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Biomechanical Aspects of Throwing a Frisbee: A Review

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Table of Contents

1. Introduction

The flying disc, more commonly known as the Frisbee^{TM}, has been used as a sports instrument more and more all over the world throughout the past few years. Disc sports in all its facets is a fast growing sport. Today, all over the world and especially in the USA there are millions of people fascinated by the unusual throwing characteristics of a Frisbee^{TM}, which are thrown at the beach, in Disc Golf, in Guts, in Dog Frisbee, by Freestylers and in Ultimate Frisbee. In all these areas, people are trying to use the special characteristics of disc trajectories. Sometimes, quite extraordinary flight paths, such as S-shaped curves, can be observed. Some athletes reach throwing distances far more than $200m$. (Bernandi, 2016) Others achieve air hangtimes of more than 16 seconds before catching the disc afterwards (Bernandi, 2016). In Ultimate Frisbee athletes need to execute their throwing routine so precisely that a running teammate can catch it easily in spite of being followed by an opponent player within one metre distance. In every discipline, there are highly experienced throwers all over the world. Every one of them has optimized his own moving programme by throwing the disc thousands of times. Therefore, this thesis will try to summarize the recent science about the biomechanical aspects of throwing a Frisbee^{TM}.

1.1 Outline

By comparing the throw of a ball and the throw of a disc, several differences can be found by just watching the movement. The purpose of this work is to present the recent knowledge about biomechanics while throwing a Frisbee^{TM}. Therefore, a look at the recent science about the physics of a flying disc will be taken in order to create a basic understanding of the throwing mechanics presented in section 2.1.

In literature, several studies about aerodynamics and gyrodynamics of a flying disc can be found. (For example: Stilley, Carstens, 1972; Pesch, 1999; Motoyama, 2002; Hummel 2003; Potts 2005; Morrison, 2005; Crowther, Potts, 2007; Baumback, 2010) The scientists used different methods of research. Some of them aim at examining the aerodynamic aspects by looking at the airflow around the disc in a wind tunnel (e.g.: Nakamura, Fukamachi, 1991; Potts, Crowther, 2000/ 2001/ 2002/ 2007). Other studies try to calculate the trajectory of the disc by having a closer look at the theoretical physical aspects or by simulating the flow around a disc with a computer programme (e.g.: Motoyama, 2002; Hummel, 2003; Potts, 2005; Morrison, 2005; Baumback, 2010). Furthermore, investigations to create a model of the trajectory of disc flight were undertaken. These models are often either compared with wind tunnel test data (Potts, 2005; Koyangi, Seo, Otha, Ohgi, 2012) or high speed video data (Hubbard, Hummel, 2000; Hummel, Hubbard, 2002). Section 2.2 gives an overview about the methods used in investigations carried out in Frisbee™ science.

In order to get a more detailed analysis of the biomechanical aspects when throwing a Frisbee, cameras are not only used to investigate flight trajectories but also to film throwers. Section 2.3 leads to the above named purpose of this work. In 1999, Robert Pesch investigated the throwing movement of several German high level Ultimate Frisbee players with video data from a regular camera. Hummel and Hubbard (2001) tried to create "*A Musculoskeletal Model for Backhand Throws"* by analysing high speed video data from high level Ultimate Frisbee players. Later, Hummel verified the results in her master thesis (Hummel, 2003). Sasakawa and Sakurai (2008) compared the throwing movement of skilled and unskilled players throwing a sidearm for distance. They also used high speed video data to evaluate the change of joint angles.

1.2 Literature Research

The scientific literature, which could be found for this review, was mostly searched on the internet with the help of two scientific search engines, "scopus" (www.scopus.com) and "google scholar" (www.scholar.google.com). The licenses from the Westfälische Wilhelms Universität - Münster enabled original texts from several scientific journals to be found. One difficulty encountered when searching biomechanical studies about the Frisbee throw is that the number of investigations is small and most of the investigations do not care about the biomechanics but the physics of a flying disc. In 2003, Sarah Hummel confirmed in her Master Thesis: "Quantitative Frisbee throw biomechanics have been neglected." Consequently, only three proper investigation projects were found.

On the 14th of December 2015, the search term "Frisbee", when typed into the scientific search engine "scopus", led to two well usable results: Hummel, Hubbard (2001) and Hummel (2003). The same search offered several other studies about flying discs, which do not discuss biomechanics but different parts of Frisbee™ physics. Most of these were used as background information for chapter 2.1. One book (Lorenz, 2006), from which an extract was found in "scopus", could be borrowed from the TU München Universitätsbibliothek. On the 15th of December 2015, the search term "Frisbee Throw" in "scopus" and "google scholar" led to no additional results regarding biomechanical studies but to a few more studies about aerodynamics and modelling Frisbee™ flight. On the same day, the search term "backhand Frisbee" presented the same sources discussing biomechanics named above, but the term "sidearm Frisbee" in "google scholar" led to the study from Sasakawa and Sakurai (2008). Later on, the search term "Frisbee" was replaced by "flying disc", in both search engines and all combinations of search terms, which did not change the results essentially. Several other search terms were used with no success. From then on, the search was based on the references of the already found sources. Thus, with the help of explicit search terms, a few more studies were found. Inter alia the dissertation of Jonathan Potts (2005), which is often quoted in chapter 2.1, was found. By enquiring with a representative of the German Frisbee Sport Verband, Ralf Simon, one more unpublished source, the thesis for diploma by Robert Pesch (1999) was found. On the 17th of March 2017, an explicit search with the title of this thesis in “google scholar” led to the same document, which had already been at hand. A search in the database of the Universitäts- und Landesbibliothek Münster did not give any suitable results.

A list of all researches found in investigations or quoted from a main source of this thesis is given in table A1 in the appendix.

1.3 Basic Definitions

In the following report, a few definitions will be used. There is an earth fixed Cartesian coordinate system with the Y-axis in throwing direction, the Z-axis upwards, and the X-axis sideways. Furthermore, there is a disc fixed Cartesian coordinate system. This system uses the z-axis as the cross product of two vectors laying in the disc plane ($\overrightarrow{CA}$ and $\overrightarrow{CB}$ in figure 1 by Sasakawa and Sakurai (2008)), the x-axis as $x = v \times z$ and the y-axis as $y = z \times x$.

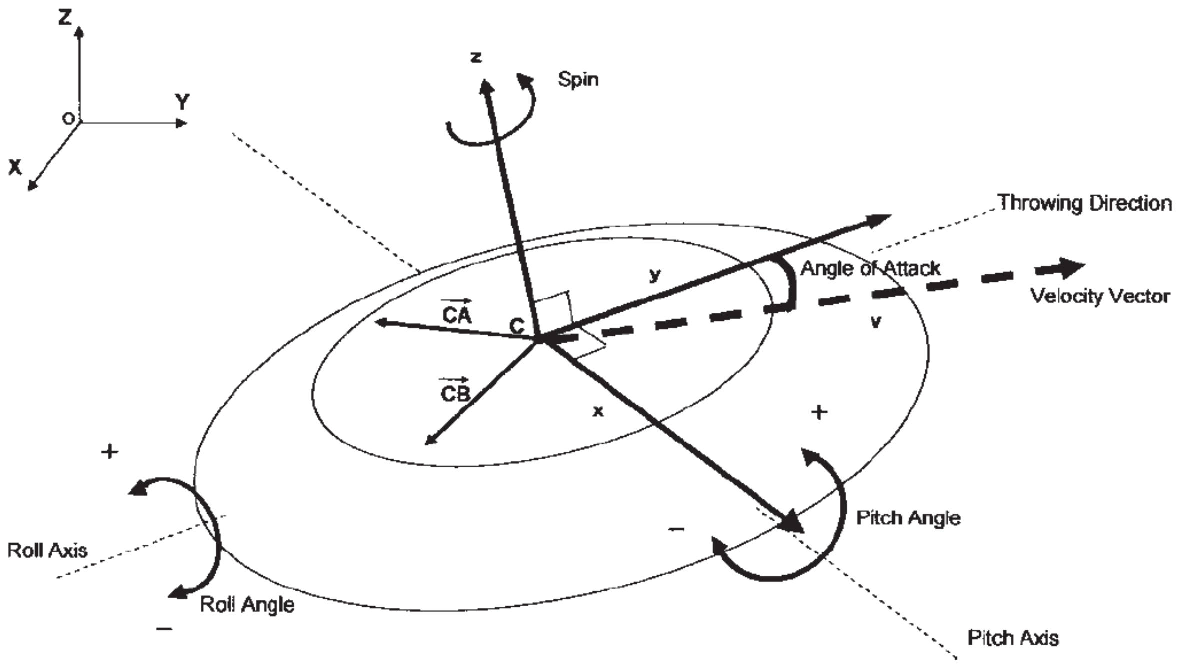

Figure 1: Definition of initial coordinate systems and important angles. Figure from Sasakawa, Sakurai (2008, p.315)

The pitch angle θ is then defined as the angle between y and Y in the YZ plane. The roll angle φ is defined as the angle between x and X in the XZ plane. Both are defined as zero for the discs plane being horizontal to the XY-plane. The angle of attack α is defined as the angle between the velocity vector v and the y-axis. Roll angle φ, pitch angle θ and angle of attack α can be calculated by the cosine equation for the scalar product:

$$\vec{a} \cdot \vec{b} = |a| \cdot |b| \cdot \cos\tau \Leftrightarrow \tau = \cos^{-1}\left(\frac{a \cdot b}{|a| \cdot |b|}\right), \text{ with } \tau \text{ as the angle between } \vec{a} \text{ and } \vec{b}$$

2. Recent Science

2.1 Flight Characteristics of a Sport Disc

2.1.1 Ballistic

Every observer of a flying disc can say that it possesses many features different to a ball. There are several physical effects influencing the flight of a disc. Lissaman and Hubbard (2010) chose the approach of investigating the flight characteristics by watching the ballistic. As a starting remark, they highlighted that the ballistic range of a throw is given by

$$R = \left(v^2/g\right) \sin 2\theta.$$

This range is maximised with a launch angle $\theta = 45^O$ and varies with the quadratic velocity v^2 (Lissaman, Hubbard, 2010). They added the effect of drag and found out that range decreased essentially. By then adding effects of the aerodynamic lift, range increased again. The following graphic from "*Maximum range of flying discs"* by Lissaman and Hubbard (2010) illustrates these effects.

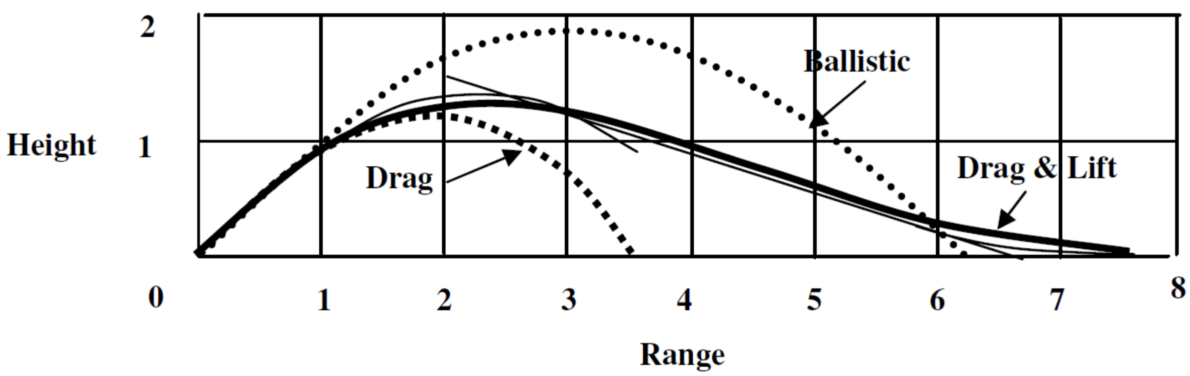

Figure 2: Trajectories of sport disc with drag and lift (Figure from Lissaman, Hubbard, 2010, p.2530)

Furthermore, they evaluated that in an ideal case a disc should be able to change its pitching angle independently to create the right angle of attack for best lift/drag ratio at every moment during the flight (Lissaman, Hubbard 2010, p.2531 f). Because of the fact that this ideal case is not real, a thrower needs to search for an optimum ascent angle to increase his range. “For the real case, the disc is released with an initial launch speed, spin rate and attitude.” (Lissaman, Hubbard 2010, p. 2532). The thrower has only these three options to influence the flight path of the disc, which is why he needs to think about a few physical aspects for finding the best release options. These aspects can be basically departed in aerodynamics and gyrodynamics.

2.1.2 Aerodynamics

In literature, two main aerodynamic effects on the flight of a Frisbee™ can be found. These are drag and lift. The only unneglectable force also influencing the disc is gravity g. Hence, there is the initial impulse of the thrower, the lift from displacing the air pushing the disc up, the drag from displacing the air, which slows the disc down, and the gravity pulling the disc to the ground. Although, Crowther and Potts (2007) tried to integrate the external factor of wind in their estimations, this factor is mostly neglected in other studies. Regarding the aim of this thesis, the aspect of wind will not be taken into consideration.

2.1.2.1 Drag

To describe the drag D, which constantly effects the flight of a disc in the opposite direction of the velocity vector, Demtröder (2013, p.228) used the following formula:

$$D = C_d A \rho v^2 \frac{1}{2}$$

with C_d as the drag coefficient, which depends on the shape of the flying object, A as the planform area (the area viewed perpendicularly from above or underneath), ρ as the density of the air and v as the relative velocity between object and fluid, here the norm of the velocity vector of the disc. Due to its asymmetric shape, the drag of a disc changes with the angle of attack α. Therefore, it is necessary to concretize the drag coefficient C_d as a function of α. Hummel (2003, p.7) suggested the following formula:

$$C_d = C_{d_0} + C_{d\alpha}(\alpha - \alpha_0)^2.$$

Here, $C_{d_0}$is the coefficient generated only by “skin friction and pressure drag” (Hummel 2003, p.8). α_0 is the angle of attack with the least drag, which Crowther and Potts (2007) measured as $\alpha_0 =$

$-2{,}97°$. $C_{d\alpha}$, the induced drag coefficient is depending quadratically on the angle of attack α. Hummel and Hubbard followed this formula in their later publications. This is basically the idea of most of the authors, who described the drag force in a more detailed manner (Morrison, 2005; Lorenz, 2006, p.176; Baumback, 2010). Potts and Crowther (2002) investigated drag and lift in wind tunnel experiments. A clear equation is only found in their later publication (Crowther, Potts, 2007), but their collected data fits very well to Hummel's equation.

Moreover, Potts (2005) plotted values for various Reynolds numbers, which are "defined as the ratio of inertial forces to viscous forces" (Hummel, 2003, p. 8), and drag/ lift coefficients. He found out that they are independent from each other for relevant angles of attack (Potts measured for $\alpha = (-10°, \ldots, 30°)$). Another remarkable aspect that Potts (2005) detected is that spin does not effect drag substantially.

The physical background of drag is based on the fluid dynamics or the flow behaviour of the air around the flying disc. Due to the focus of this thesis, these aspects will not be discussed any further.

2.1.2.2 Lift

Analogous, it can be found that the lift L, which effects the disc perpendicular to the velocity vector, "roughly along the positive Z-axis" (Hummel, 2003, p.9) of the earth fixed Cartesian coordinate system, is given by

$$L = C_l A p v^2 \frac{1}{2}$$

(Hubbard, Hummel, 2000, p.3). Here C_l is the lift coefficient. Again, it depends on the shape of the object gliding through a fluid. The shape relative to the velocity vector changes with the angle of attack. At this point, Hubbard and Hummel (2000, p.3) do not give a quadratic but a linear correlation:

$$C_l = C_{l_0} + C_{l\alpha}\,\alpha.$$

C_{l_0} is the lift coefficient at $\alpha = 0$ and $C_{l\alpha}$ is the slope. The lift coefficient C_l is zero at $\alpha = \alpha_0$, where the drag is least (Hummel, 2003, p.11). Lorenz (2005), Morrison (2005), Crowther & Potts (2007) and Baumback (2010) follow Hubbard's and Hummel's equations.

The disc splits the streaming air in two separate airstreams. At a slightly positive angle of attack, the trailing edge forces the incoming air downwards, which causes the disc, in accordance to Newton's third law, to go upwards. On the upper side of the disc, the air tends to follow the shape of the disc and goes down, when $\alpha > 0$. Referring to Newton, the disc forces the air downwards and therefore the air forces the disc upwards.

Looking at the shape of the disc, effects can be found, which force the air flowing over the upper side faster than the air underneath the disc. Bernoulli's equation states that higher velocity goes ahead with lower pressure and vice versa. This difference in pressure induces lift. More detailed information about the physical background for lift can be found in Demtröder (2013, p.230 f) and Hummel (2003, p.9 f). In this thesis, it will not be discussed any further.

Crowther and Potts (2001, 2002, 2007) and Potts (2005) measured the coefficients in wind tunnel experiments and found out that for relevant angles of attack, the quadratic model in drag coefficients and the linear model in lift coefficients fits well. However, they have a small variation in the drag coefficient for higher angles of attack.

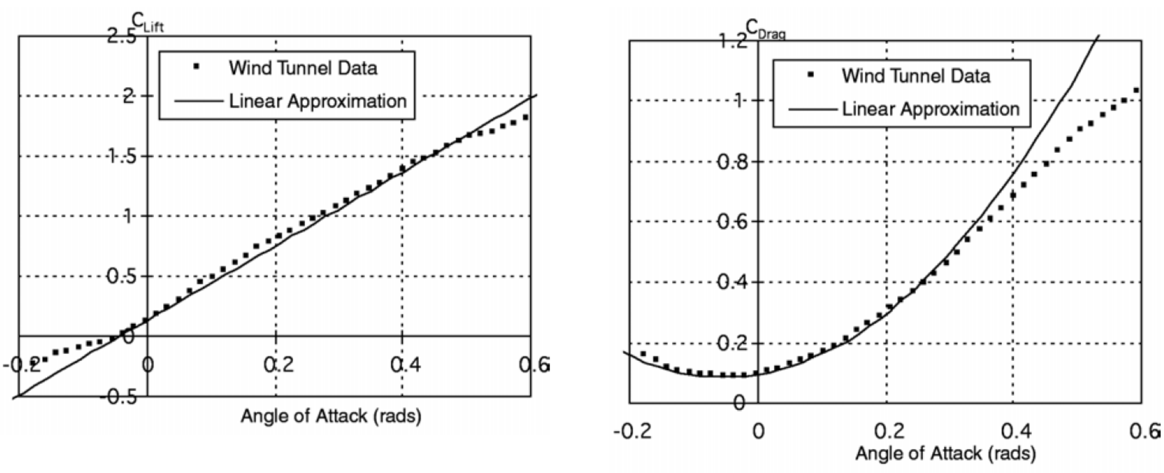

Figure 3: Lift coefficients (left) and drag coefficients (right) (Figure from Potts, Crowther, 2007, p.9). Approximation from Hubbard and Hummel and wind tunnel data measured from Crowther and Potts (2007, p.9). On the right, the term linear should be replaced by quadratic.

2.1.2.3 Consequences of Drag and Lift for the Thrower

Once a thrower knows about drag and lift, he knows about the angle of attack he can choose. If he throws the disc with a low angle of attack, the disc will have low drag but low lift. The flight path will be fast and flat, possibly travelling for a long range, depending on other influencing factors. If the thrower chooses a higher angle of attack, the disc will experience higher drag and higher lift, which means loosing speed faster but staying in the air longer. Throws like that are normally easier to catch, but they do not reach long ranges (See section 2.1.4).

2.1.3 Spin and Gyrodynamics

2.1.3.1 Roll Moment, Pitch Moment and Spin

As every free body in the air, the disc has six degrees of freedom. Three degrees in translational movement and three in rotation around the axis of the disc fixed coordinate system. During flight, torques and angular momentums appear, which change the roll angle, the pitch angle and the angle of attack in principal. The roll moment is a change of the roll angle velocity and therefore an accelerated rotation around the y-axis. Its value is positive when, out of the thrower's perspective, the right side of the disc is moving up. The pitch moment is a change of angular velocity in the pitch angle and can be expressed as an accelerated rotation around the x-axis. Its value is positive when the nose is facing upwards. In aerodynamics, roll, pitch and yaw angles can be found. In this case, the yaw moment is identified by the spin of the disc. Or, to be precise, the yaw moment appears in the throwing movement when the throwers give spin to the disc. Afterwards, the spin is the angular velocity around the vertical yaw-axis. Viewed from above, a disc thrown with a right-handed backhand spins, as defined, clockwise. Thrown with a right-handed sidearm (the upper body/limb movement is roughly comparable to a sidearm pitch in baseball (Sasakawa, Sakurai, 2008, p.319)), it spins anticlockwise. Here, the anticlockwise rotation is defined as positive.

2.1.3.2 Gyroscopic Effects

Gravity is acting at the centre of mass (COM) and pulls the disc in negative Z-direction. Lift is acting at the centre of pressure (COP) in "roughly positive" Z-direction (Hummel, 2003, p.9).[1] Regarding the physical background of lift, the COM is not necessarily identical to the COP (Hummel, 2003, p.11f). In fact, the COP depends on different variables, especially the angle of attack, and therefore moves during flight. "A simple physical explanation for why the COP behaves as it does is not available in terms of fluid dynamic principles, but it relates to the shape of and flow around the Frisbee™" (Hummel, 2003, p.14).

Regardless of why the COP moves, it does move and therefore creates a pitch moment owing to its displacement to the COM. If the COP is ahead of the COM, it will result in "a nose up pitching moment" (Potts, 2005, p.39) and vice versa. However, a rotating disc is influenced by gyroscopic effects. In this case, these effects provoke that the pitch moment is turned into a roll moment by precession. Lissaman and Hubbard (2010, p.2532) summarize: "Discs are normally unstable due to displacement of the center of mass, and, to avoid tumbling, must be stabilized by spin. The pitching

[1] Lift is actually acting vertical to the airflow or the velocity vector. (See Demtröder (2013, p.231)).

moment couples gyroscopically with the spin to induce a roll rate." In accordance to the definition of the disc fixed coordinate system, a negative spin rate (thrown with a right-handed backhand) and a negative pitch moment (angle of attack $\alpha < 9°$) induce a roll right wing down (Hummel, 2003, p.20).

"In general, a pitching moment causes a precessional roll rate and roll moment causes a precessional pitch rate" (Hummel, 2003, p.20). Following Potts (2005, p.144), there is no significant roll moment for $-10° < \alpha < 30°$. Thus, the only noticeable aerodynamic torque acting on the disc is the pitch moment with its precessional change of roll rate.

Hummel and Hubbard (2000, 2002), Potts and Crowther (2002), Hummel (2003) and Potts (2005) gave detailed equations for roll and pitch moment with further coefficients. In view of the aim of this work, these are not referenced to explicitly at this point. To get an impression of the connection between the angle of attack and the pitching moment, the following graphic from Crowther and Potts (2007, p.9) is shown. They calculated and measured data for the pitch moment coefficient depending on the angle of attack. The linear approximation is also found in Hummel and Hubbard (2000, 2002) and Hummel (2003).

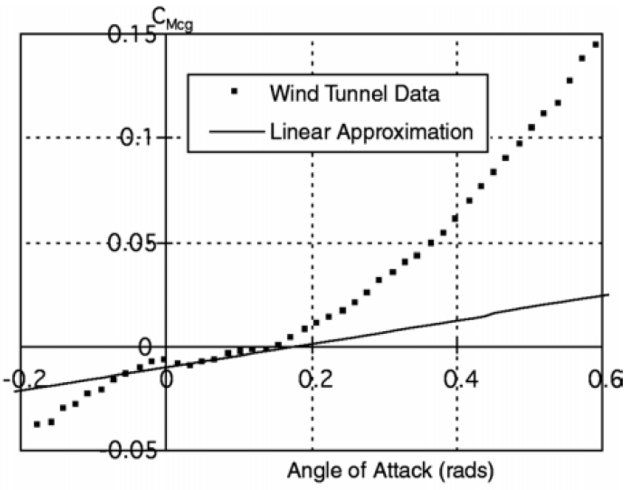

Figure 4: Pitching moment measured by Potts and Crowther (2007) and the linear approximation of the flight model Hummel and Hubbard used. (Figure from Crowther, Potts, 2007, p.9)

2.1.3.3 Consequences of Gyroscopic Effects for a Thrower

The Precession effect can be observed in long distance throws, in which the disc usually does not flip over, but has a tendency to roll left or right during the flight, depending on the spin direction. A good thrower knows about this effect and tries to launch the disc with a compensating roll angle. Hence, Lissaman and Hubbard (2010, p.2532) state that a disc should "be launched at the correct ascentangle, usually with a pronounced bank [roll angle, remark by the author], calling for skill by the thrower."

Due to the change of the velocity vector and angle of attack respectively, the pitching moment can change significantly during flight. If the angle of attack passes 9°, the sign of the pitching moment coefficient will change. In other words, the induced roll rate changes direction. This is a reason why sometimes S-shaped flight paths can be observed.

"Keeping the precession down to a few degrees over the flight duration of a couple of seconds is all that is needed" (Lorenz, 2006, p.175). Lorenz comes to this conclusion as an answer to how to achieve a good throw. He gives the explanation that the "precession rate is equal to the pitch moment divided by the moment of inertia and spin angular velocity" (Lorenz, 2006, p.175). The moment of inertia depends on mass distribution and the rotation axis. Both are predetermined by the construction of the disc and the way of throwing respectively. An Ultimate Frisbee disc has a relative thin plane and a relative deep and thick lip to create a high moment of inertia.[2] A thrower cannot influence the pitch moment, but he can influence the spin angular velocity. Therefore, he will try to throw the disc with high spin, because a high spin rate induces a low roll moment. For a more detailed explanation see Hummel (2003, p.18 ff).

Due to the increased drag resulting from the oscillation in the angle of attack, a thrower tries to throw the disc in a way with least wobble. That means, he tries to throw the disc without "any components of angular velocity about the x and y-axes" (Hummel, 2003, p. 18). If he does, a significant wobble will be observable in the beginning of the throw. However, the wobble dies out with the first metres of flight due to aerodynamic effects. Hummel (2003, p.19) proved that a once wobbling flying disc would not stop wobbling until a kind of torque acts on the disc. Without any detailed information she names the aerodynamic effects, which induces a pitch moment and the precessional roll rate explained above, as the wobble decreasing torque (Hummel, 2003, p.19).

2.1.4 The Trajectory of a Flying Disc

2.1.4.1 The Ideal Trajectory

The trajectory of an ideal Frisbee™ throw can be departed in three phases. The ascent from release until reaching the apex, a gliding phase and a so called "flare out" (Lissaman, Hubbard, 2010, p.2531). To achieve a theoretical ideal throw, the disc would have to take a pitch angle for the least drag and

[2] The lip of a Disc Golf disc is not deep but thicker. It is technically harder to throw it, but it flies with an even less precessional roll rate.

the most lift. It would ascend in a parabola like a ballistic throw. Starting with a pitch angle of roughly 45°, it would change its pitch angle for the least drag during climb. At the highest point it would start to lose height and change the pitch angle for optimal lift / drag ratio. This phase is called the “gliding phase”. At the end, when velocity comes close to zero, the pitch angle would increase again to achieve an optimal angle of attack and lift respectively (Lissaman, Hubbard, 2010, p. 2532).

At the moment, discs changing their pitch angle by themselves are science fiction. Therefore, throwers try to throw in a way, which yields an optimal gliding phase. Regarding precession effects, Lissaman and Hubbard (2010, p.2532) point out: “Long range is achieved by exploiting the gyroscopic terms so that the disc acquires a roughly wings level state near the apex and thereafter glides approximately constant attitude, giving the optimum lift/drag ratio.”

Hence, the morphological characteristics of the ideal gliding phase are comparable to birds flying without flapping or gliders in the air. This means the disc flies wings level or with the roll angle $\varphi = 0°$. An optimal gliding phase is achieved by flying in an optimal lift/drag ratio as long as possible.

2.1.4.2 Free Disc Trajectory

It is possible to throw the disc over far distances, up to 20m or more, in a nearly horizontal or nearly wings level trajectory (Hummel, 2003, p.21). Due to the spin of a disc, a precessional roll rate appears at all angle of attacks except for approximately $\alpha = 9°$ (compare 2.1.3.2). Thus, to stay straight, the disc needs to fly with $\alpha = 9°$ during the whole flight. Caused by drag, the disc loses velocity during its flight. As a consequence, the lift decreases causing the velocity vector slowly turning to point towards the earth surface. With this turn, the angle of attack increases again. Hence, it is possible to produce a throw with an almost constant angle of attack of $\alpha = 9°$ and therefore no precession but a so called “downward steady glide” (Hummel, 2003, p.22 f). Hummel (2003, p. 24) calculated the optimal values for staying straight at $\alpha = 9°$. These are a pitch angle of $\theta = -10.3°$ and a straight velocity $v = 9.1\frac{m}{s}$.[3] Due to the gyroscopic effects, it is not possible to throw a disc exactly horizontal, which means without derivation in X- and Z-values. The pitch moment induces a precessional roll rate and a roll angle $\varphi \neq 0°$ produces non-zero elements in the X-axis components of the lift vector. However, the thrower can try to expand the phase of straight horizontal movement by a high velocity, an angle of attack with low drag and a high spin rate. As long as the angle of attack does not change too much, (look at the plateau in the pitching moment coefficient Crowther and Potts

[3] In this case, gravity g accelerates the disc and exactly compensates the drag effect causing a constant velocity. This kind of throw is of theoretical interest and not useful.

(2007) measured) and the spin rate is high, precession stays low (compare 2.1.3.3). Due to the plateau in the pitching moment coefficient, an angle of attack with gravitation balancing lift is possible. In this case, a small but noticeable roll rate appears. This kind of throw is an often used throw for precise passing in Ultimate Frisbee.

Hummel (2003, p.24 f.) gives common throw conditions for a usual 25m flight by an experienced thrower. She developed a computer simulation of a flight and compared the results to high speed video data (see below). This example with "little initial wobble" (Hummel, 2003, p.24) starts at a velocity $v = 14\frac{m}{s}$, a spin rate $r = 50\frac{rad}{s}$, a pitch angle $\theta = 11°$, a roll angle $\varphi = 0°$ and an angle of attack $\alpha = 5°$ (Hummel, 2003, p.24 f.). Therefore, the velocity vector points slightly upwards with $\beta = 6°$ to the horizontal.

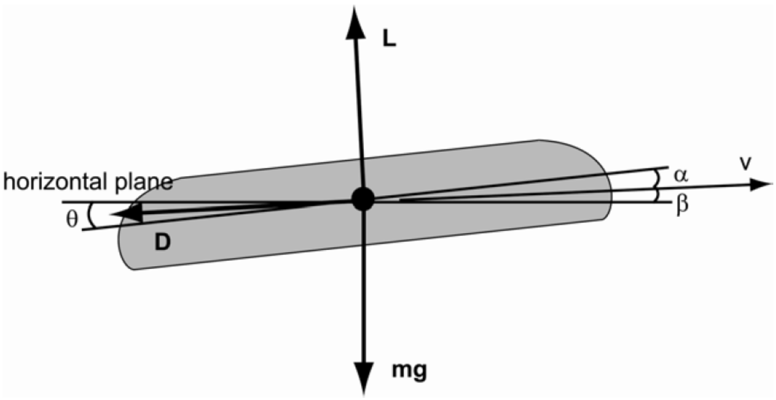

Figure 5: "Force and flight configuration for $\alpha = 5°, \theta = 11°$ (Figure from Hummel, 2003, p.25).

Due to the low angle (below 9°) of attack, the disc immediately starts to curve right. During the flight, drag slows the disc down to less than $4\frac{m}{s}$. Thus, the lift decreases and the downward component of the velocity vector increases. Hence, the angle of attack increases, which is why the lift is heightened again and the disc does not sink as fast as it would without the effect of a growing angle of attack. Simultaneously, the increasing angle of attack couples gyroscopically with the roll angle and the disc curves to the left at the end of the flight (Hummel, 2003, p.24 ff.). During its $3.5sec$ of flight, the disc moves $25m$ in Y direction and approximately $0.8m$ in X direction. The greatest vertical distance to the Y-axis was a height of $2.55m$ after $1.5sec$ and nearly $1m$ to the right after $3sec$ of flight. The pitch angle changed less than 2° during the flight, but the angle of attack increased after $1/4$sec of decreasing nearly constantly. Hence, the disc ascended while the angle of attack decreased and sunk while the angle of attack increased.

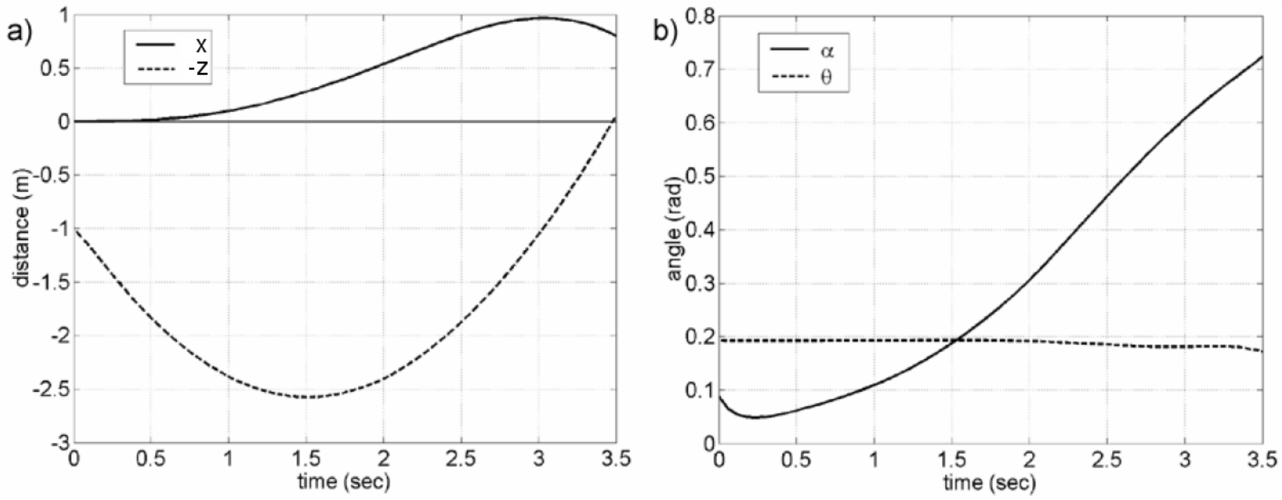

Figure 6 a: X and -Z position of the COM during the flight (Figure from Hummel, 2003, p.26). Due to the definition of the earth fixed Cartesian coordinate System, the coordinate axes are renamed here.

Figure 6 b: Angle of attack α and pitch angle θ during flight (Figure from Hummel, 2003, p.26).

2.1.4.3 Special Phases of the Flight

The most important phase for reaching distance is the gliding phase, as discussed above. "Long range is achieved by exploiting the gyroscopic terms so that the disc acquires a roughly wings level state near the apex and thereafter glides approximately at a constant attitude, giving the optimum lift/drag ratio" (Lissaman, Hubbard, 2010, p.2532). This quotation from Lissaman and Hubbard out of "Maximum range of flying discs" gives reasons for the aim of the ascent phase. If the aim is range, the disc will need to rise as high as possible, but more importantly, it needs to be given an optimum pitch and roll angle for an effective gliding phase.

At the end of a flight a so called "flare out" can be observed, which pilots from aircrafts are familiar with. It gives a name to the phenomenon of increasing lift at the expense of forward speed when the disc or the aircraft flies close to the ground (Lissaman, Hubbard, p.2530). At the end of a flight, it is sometimes visible by a disc slowing down to $v_Y = 0\ \frac{m}{s}$ and a high precessional role rate.

2.2 Methods in Frisbee™ Flight Science

To investigate the physics of a Frisbee™ flight, most scientists used wind tunnel measurements. Lorenz (2005) used on board measurements and biomechanics were scrutinized with high speed cameras by Hummel (2001, 2003) and Sasakawa and Sakurai (2008). At the beginning of her Master Thesis, Sarah Hummel points out: "Until recently scientific, quantitative research on Frisbee flight mechanics was relatively scarce" (Hummel, 2003, p.3). In table 1 (see appendix), which lists all scientific publications at hand or research quoted by these, only four publications older than 20 years paying attention to a flying disc can be found. Over the last eighteen years, the scientific interest in Frisbee flight dynamics has increased enormously.

The first investigation of Frisbee^{TM} dynamics, which is quoted in the available material, is those from Stilley and Carstens (1972, quoted from Potts, 2005, p.44), who measured drag and lift with and without the influence of spin in a wind tunnel. There are a few older sources looking for the aerodynamics of discus or other flying discs (compare Potts, 2005, p.67). Due to the focus of this thesis, these are neglected. In 1980, Lazzara, Schweitzer and Toscano (quoted from Potts, 2005, p.45) and in 1998 Ali (quoted from Potts, 2005, p.45) measured lift and drag in wind tunnels. In 1991, Nakamura and Fukamachi were the first investigating the airflow.

With the end of the last millennium, methods began to become more versatile. Pesch (1999) tried to find out from ten very successful German Ultimate Frisbee players which factors or conditions are important for a long backhand throw. He utilized a questionnaire and a regular camera. In 2000, Hubbard and Hummel published "Simulation of Frisbee Flight", which was a computed estimation of a Frisbee^{TM} flight trajectory compared with high speed cameras. Potts and Crowther started to publicize several wind tunnel measurements in 2001. These last two research groups left their mark on the science about Frisbee^{TM} characteristics during the last years and often quoted each other. There are a few scientists calculating theoretical mathematical models of Frisbee flights without measured data. Lorenz (2005) was the only one making investigations with on board sensors in recent science.

Sarah Hummel wrote in her Master Thesis: "Quantitative Frisbee throw biomechanics have been neglected" (Hummel, 2003, p.2). Her own publications two years earlier (Hummel, Hubbard, 2001) were the first paying quantitative attention to the biomechanics. In her Master Thesis from 2003, she expanded the researches (compare 2.3.2). Controneo (1980, quoted from Hummel, 2003, p.2) made some comparing research between backhand and forearm throws regarding the force contributions. Sasakawa and Sakurai (2008) published a research about the differences in the forehand throwing motion between skilled and unskilled players (compare 2.3.3).

2.2.2 Wind Tunnel Measurements

Apart from the works of Stilley and Carstens (1972, quoted from Potts, 2005, p.44), further investigations on lift and drag in wind tunnel measurements were done by Lazzara et al (1980, quoted from Potts, 2005, p.45), Ali (1998, quoted from Potts, 2005, p.45), Nakamura and Fukamachi (1991), Mitchell (1999, quoted from Potts, 2005, p.45), Yasuda (1999, quoted from Potts, 2005, p.47), Potts and Crowther (2000a, 2000b, 2001a, 2001b, 2002, 2007), Higuchi, Goto, Hiramoto and Meisel (2000, quoted from Potts, 2005, p.46), Potts (2005) and Koyanagi, Seo, Otha, Ohgi (2012). After analyse of a number of publications, it can be noticed that the wind tunnel is the main instrument in the investigation of flying discs. As already mentioned, the named scientists mostly measured lift and drag.

Gyrodynamics and effects from spin respectively, were researched by Stilley and Carstens (1972), Lazzara et al (1980), Yasuda (1999), Potts, Crowther (2000a, 2002, 2007) and Potts (2005). Stilley and Carstens found “spin to be negligible” (Potts, 2005, p.45). “Lazzara et al concluded that spin generates a small lift component” (Potts, 2005, p.45). The recent opinion of Potts and Crowther (2002) and Hummel (2003) considers spin as not influencing the flight dynamic per Magnus effect significantly (Crowther, Potts, 2002, p.6), but the influence of spin on the flight dynamics is given by the gyroscopic effects described above.

Mitchell (1999, quoted from Potts, 2005, p.45) and Potts (2005, p.63 ff.) investigated if the flying characteristics of a disc depend on the Reynolds number. Both of them came to opposite conclusions. While Mitchell discovered a strong interdependence between the Reynolds number and flight characteristics, Potts and Crowther found out that for a speed range from $6\frac{m}{s}$ to $25\ \frac{m}{s}$ and an angle of attack between $-10°$ and $30°$ “force and moment coefficients are approximately independent of Reynolds number” (Potts, 2005, p.64 or compare Potts, Crowther, 2002, p.5). Hummel (2003) refers to Potts and Crowther (2002) and does not investigate the influence of the Reynolds number herself.

A few scientists tried to investigate the airflow around the disc in a wind tunnel. Nakamura and Fukamachi were the first, who visualized the airflow around a flying disc. They ascertained “that a pair of longitudinal vortices” (Nakamura, Fukamachi, 1991, p.35) are formed behind the flying disc. These vortices are rotating inside down and produce a downwash. Hence, a lifting force on the flying disc appears (Nakamura, Fukamachi, 1991, p.35). Potts and Crowther (2000a, 2000b) also found this pair of vortices. They added several visualisations of surface flow on different angles of attack and different velocities. With particle image velocimetry on the airflow over a Disc Golf disc, Higuchi, Goto, Hiramoto and Meisel (2000 quoted form Potts, 2005, p. 46) added another type of disc being investigated. They focused on the vortices. Later, wind tunnel investigations were used as comparing data for computer simulations (Koyanagi, Seo, Otha, Oghi, 2012; Lukes, Hart, Potts, Haake, 2014).

Probably, due to the setup of wind tunnel investigations, biomechanical aspects were neglected.

2.2.3 Computer Simulations

A few computer simulations can be found in literature. Most of them were estimated with MATLAB (Hummel, Hubbard, 2000, 2002; Hummel, 2003; Potts, 2005). Crowther and Potts (2007) and Koyanagi et al (2012) developed their own mathematical models. All of these publications tried to present a model, which is able to calculate the trajectory of a flying disc by given start conditions.

Potts (2005), Crowther and Potts (2007) and Koyanagi et al (2012) compared their estimations of flight parameters, such as lift and drag or the velocity on the global coordinate system, with wind tunnel data. For validation of their results they needed additional data from real flight situations. Therefore, Potts (2005) and Crowther and Potts (2007) used data from Hummel (2003). They found out that their estimations were "qualitatively similar" (Crowther, Potts, 2007, p.12) but "the velocity magnitude for the simulated data shows a rapid decrease immediately following launch that is not present in the experimental data" (Crowther, Potts, 2007, p.12). They suspected that their own calculated data showed this rapid decrease of velocity values due to wobbling in the beginning of the flight. This wobbling in the throws used as reference for an iterative MATLAB algorithm induced an oscillation on the angle of attack and hence increased drag significantly. Koyanagi et al (2012) measured their own real flight data with a motion capture system they did not explain in detail.

Hummel and Hubbard (2000, 2002) and Hummel (2003) used high speed cameras (120 Hz in 2000, 120 Hz and 200 Hz in 2003)) and markers on the disc to collect data for their estimations. In their first publication (2000), a mathematical model with eight aerodynamic coefficients, which have been iteratively approximated with the collected flight data, was presented. In Hummel and Hubbard (2002), two further coefficients were added to in sum ten coefficients. They wrote a MATLAB algorithm for the determination of the parameters for each flight. For her Master Thesis, Hummel (2003) summarized the results from the two prior investigations and added a few optimizations.

In 2014, Lukes, Hart, Potts and Haake investigated the flow around a disc with a computational fluid dynamics (CFD) simulation. This later publication in Frisbee™ flight dynamics gave a model for describing the airflow over a rotating disc projectile. Lukes et al (2014) compared CFD analysis results with experimental results from the wind tunnel to optimize the flow model.

2.2.4 On Board Measurement

The only scientist investigating flight dynamics with on board measurement is Ralph Lorenz (2005). In a first test phase, he placed a microcontroller, an accelerometer and two button cells underneath the disc. In a second and third phase, he added several other sensors.

Lorenz (2005) assures that the sensors, microcontroller and batteries were placed in a way that the change in position of the COM, airflow, weight and moment of inertia got minimized (Lorenz, 2005, p.739 f). He did not explain this procedure in detail. In addition to the on board measurement, he used conventional cameras and defined coordinate systems to describe the flight path. With the help of video data, he was able to combine the data from the sensors to actual positions and attitudes of the disc in the air.

Sensor	Dimensions (cm)	Mass (g)	Type	Current	Range	Output
Pressure	1.0 × 1.0 × 0.4*	2	Piezoresistive FPX-014	2 mA	−0.25 to 1.25 kPa	Analogue 0.25–4.25 V
Magnetometer	0.6 dia × 2.5 long	4	Fluxgate FGM-1	8 mA	−50 to +50 nT	Square 20–80 kHz
Accelerometer	0.3 × 0.8 × 1.5	1	Micromachined ADXL210	0.5 mA	−10 to +10g	1 kHz PWM
Accelerometer	0.3 × 1.2 × 1.5	1	Micromachined ADXL202	0.5 mA	−2 to +2g	1 kHz PWM
Solar		<0.1	Photodiode	~1 mA	0–1350 W m^{-2}	Analogue 0–4 V
IR ranger		4	IR spot GPD12	30 mA	0.2 to 2 m	Analogue 0–2.5 V
Sound speed	2.5 × 1.5 × 1	6	40 kHz SRF04	30 mA		Square pulse
Sonar ranger	2.5 × 1.5 × 1	6	40 kHz SRF08	30 mA	0 to 6 m	Serial integer
Microphone		<1	Electret	~2 mA		Pulse rate

* Sensor body only—pressure part extends a further 8 mm.

Figure 7: Plotted table from Lorenz (2005, p.741). The sensors he used are listed. The total mass of the disc in the second test phase was $260g$*, the mass of the original disc is* $175g$.

The results received from his investigations varied in its usability. Due to the increased mass and the large number of instruments, the results were “slightly lower than ideal” (Lorenz, 2005, p.741). Another small problem Lorenz noted: “The accelerometers are over-ranged (span is $\pm 2g$) at launch and impact” (Lorenz, 2005, p.742). However, in the plotted curves of, inter alia, sun sensors or magnetometers, every rotation is countable. He measured data at 8 am “when the sun was sufficiently high above the horizon to give a good signal, but was still well in the east” (Lorenz, 2005, p.744). In combination with magnetometers depending on the magnetic field of the earth, Lorenz was able to calculate the attitude for every moment of flight, including the launch of the disc. (Lorenz, 2005, p.744). He concluded that roughly half of the launch speed and the spin is almost entirely generated in the last $0.1s$ before release. This result confirms previous finding by Hummel and Hubbard (2001) or Hummel (2003).

2.2.5 High Speed Camera

Next to wind tunnel measurements, the use of high speed cameras seems to be the second basic method when investigating the dynamics of a Frisbee™ flight. Due to the prevailing setup of research in a wind tunnel, it is, inter alia, easier to measure drag and lift coefficients or the flow over the disc.

However, with high speed cameras it is possible to investigate the whole throw, from throwing movement until impact, in a real flight. Pesch (1999), Hummel and Hubbard (2000, 2001, 2002), Hummel (2003) and Sasakawa and Sakurai (2008) used video data for their estimations.

On the one hand, high speed cameras are used to collect data for, inter alia iterative, computer estimations. Hummel and Hubbard (2000; 2002) and Hummel (2003) used high speed cameras to collect data from active LED or reflective markers as reference for their MATLAB flight model of a flying disc. This data was also used by Potts (2005) and Crowther and Potts (2007) to control their mathematical Frisbee™ flight models. On the other hand, they are used to describe the biomechanical aspects of the throwing motion from an outer and analysing view in detail. Only three projects doing this kind of research are available: Pesch (1999), Hummel and Hubbard (2001) / Hummel (2003) and Sasakawa and Sakurai (2008). These investigations will be discussed in chapter 2.3. The Super-8 Cameras Pesch (1999) used with 25 full- and 50 half pictures per second are no high speed cameras, but they were used in a comparable way to the high speed cameras in the investigations of Hummel and Hubbard (2001) ($180\ Hz$), Hummel (2003) ($120Hz$ and $200Hz$ for flight investigations (short and long flights) and $180Hz$ for investigations of the throwing movement) and Sasakawa and Sakurai (2008) ($250\ Hz$).

2.3 Review of Biomechanical Analyses of Throwing a Frisbee

There are only a few research projects, which discuss the Frisbee™ throw focusing on biomechanics. Cotroneo (1980, quoted from Hummel, 2003, p.2) and Pesch (1999) (see below) graduated from University with biomechanical studies about throwing a Frisbee™ for distance. The Master Thesis "Biomechanical and aerodynamical aspects of the backhand and sidearm Frisbee-disc throws for distance" by P.W. Cotroneo, written in 1980 at California State University, is unpublished and not at hand. Hummel (2003, p.2) refers to Cotroneo in only one sentence saying that Controneo compared the force contribution in several body segments during backhand and forehand Frisbee™ throws. Nothing else about him or his work can be found in the literature. In 2001, Sarah Hummel and Mont Hubbard published "A Musculoskeletal Model for the Backhand Frisbee Throw". This investigation was presented in Hummel's Master Thesis in a more detailed manner in 2003. The latest research, which can be found in the literature, is a "Biomechanical analysis of the sidearm throwing motion for distance of a flying disc: A comparison of skilled and unskilled Ultimate players" by Kei Sasakawa and Shinji Sakurai from 2008.

2.3.1 Robert Pesch (1999)

The thesis for diploma *„Technikanalyse des weiten Rückhandwurfes im Ultimate Frisbee – Eine empirische Untersuchung zur Strukturierung und zur Identifikation von Einflussgrößen mit Ableitung von konkreten Bewegungsanweisungen“* by Robert Pesch (1999) is unpublished but at hand. Robert Pesch investigated the long backhand throw for his diploma at the Johannes Gutenberg Universität Mainz. He chose ten German high level Ultimate Frisbee players for his investigations and combined the results from questionnaires and interviews with video data from the players for the analysis.

A substantial difference to the recent works with high speed cameras is that Pesch (1999) did not work with markers but with a qualitative description and the computer programme "Simi Motion". In this software, the position, which is marked with an active LED in latest techniques, needs to be marked manually at the computer for each picture after filming (Pesch, 1999, p.54). When this time-consuming work is done, the programme is able to calculate positions, angles and velocities between the marked points. Therefore, in Pesch's (1999) work, several systematic mistakes have been unpreventable. Due to the resolution of $640x480$ dots in "Simi Motion", the real position of a marked point could only be marked with a mistake of $\pm 0.4cm$. The, in comparison to high speed techniques, low frequency of pictures caused that points, which were moving with $20\frac{m}{s}$, aroused expanded to $40cm$ on a picture. Thus, the later marking of points in video data needed to be precise, but in spite of working with the greatest care, Pesch guessed his mistake in marking at $\pm 3cm$ per marked point in each picture. (Pesch, 1999, p.55 ff.) Pesch knew about these mistakes and saw potential for future work, which needs to reduce these mistakes (Pesch, 1999, p.59 f.).

In his questionnaires and interviews, Pesch asked about tactical and technical topics. The tactical part will be neglected completely, because it leads away from the aim of this thesis. Pesch used the answers on technical topics in combination with the filmed material to conclude a morphologic description of the throwing movement (Pesch, 1999, p.65 ff.). These aspects are of main interest.

Pesch (1999) used the results from Simi Motion for a statistical comparison between the throwers. For each variable, such as initial speed, range of the throw, angular velocities or distances that a point of the body moved during the throwing movement, he identified the median, the standard deviation, the maximum / minimum and the ideal value. The ideal values for the different variables were given by the throw with the highest throw range. The first values he listed are the velocity at launch, the slope of the forearm, the roll and pitch angle, the angle between the projection of the initial velocity vector in the XY-Plane and the Y-axis and the angle of attack (Pesch, 1999, p.73). In the further course, he discussed each value in detail (Pesch, 1999, p.73 ff.). Afterwards, he gave the same table for relevant variables in different fixed situations during the throwing movement. Pesch defined necessary variables in maximal swinging back position and in launch position. In addition, he gave values for the phases of movement between the fixed positions. The variables he defined here are normally covered distances or angles for different reference points on the body. For most of the values, Pesch (1999) defined if a thrower trying to improve his performance should maximise or optimize this value (Pesch, 1999, p.72).

Apart from tables, Pesch gave curves for several variables. Due to the limited scale, they will not be named in detail, but partly in comparison to the results from Hummel in the discussion. Figures are given in the appendix (see figures A1 and A2).

2.3.2 Sarah Hummel & Mont Hubbard (2001) and Sarah Hummel (2003)

The only published biomechanical research projects about the backhand Frisbee™ throwing movement was given by Sarah Hummel and Mont Hubbard in 2001. They conducted research on the throwing motion of high level US Ultimate Frisbee players. In her Master Thesis, Hummel (2003) presented more detailed information about these investigations.

2.3.2.1 Setup of Research

The subjects were equipped with reflective markers at the torso, the humerus, the forearm, the hand and the disc. They were asked to do backhand throws for maximum range while being filmed with four $180Hz$ cameras. However, the later analysis of data was based on seven throws of one subject with on average 57% of effort. The marker positions were chosen to collect data for a so called "musculoskeletal model" with six degrees of freedom (DOFs) of a backhand Frisbee™ throw (Hummel, 2001, p.2). The developed model is based on an over arm throwing model by Cote (2001, quoted from Hummel and Hubbard, 2001, p.2) and Cote and Hubbard (2003, quoted from Hummel, 2003, p.51) respectively, with several modifications. "The Frisbee™ throwing model has six DOFs and seven rigid bodies, the torso, clavicle, scapula, humerus, ulna, radius, and hand/disc" (Hummel, 2003, p.52). The position of the shoulder is given by clavicular and scapular motion and therefore the three translational DOFs of the glenohumeral joint are neglected. The six DOFs in the model are three rotations of the humerus, elbow flexion, pronation / supination and wrist flexion. The following figures by Hummel (2003) present the definition of the angles for the used throwing model.

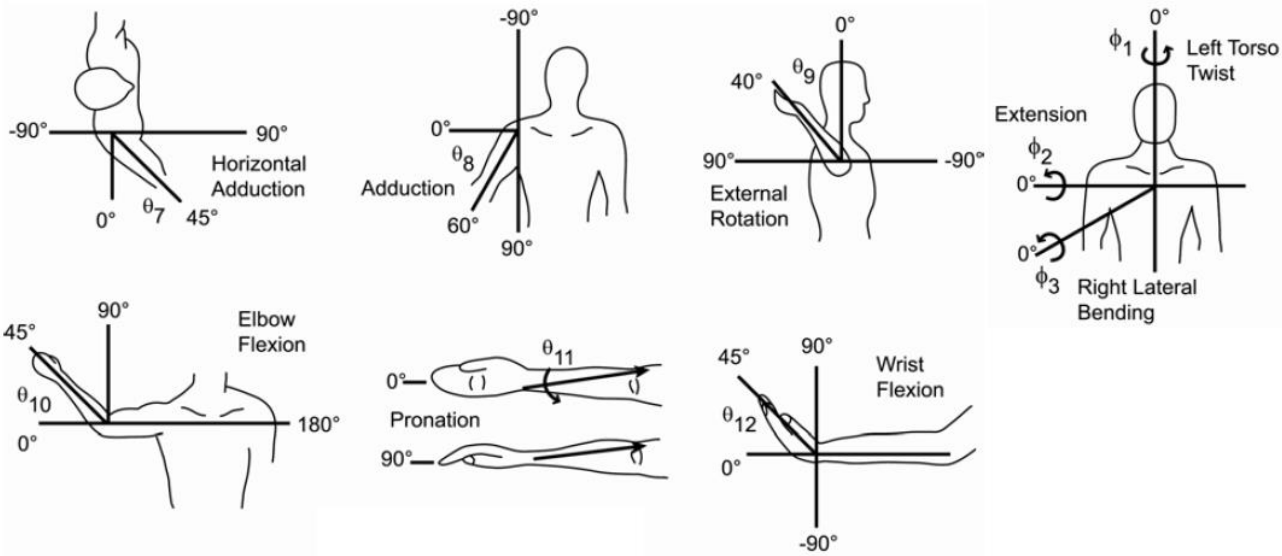

Figure 8: Used body angles. In figure 9 and in the text of her Thesis, Hummel used ϕ_2 and ϕ_3 the other way around (Figure from Hummel, 2003, p.53).

Segment	**Axis**	**Angle**	**Rotation Description**		
			Positive angles	*Negative angles*	
torso	z_T	ϕ_1	left torso twist	right torso twist	
	y_T	ϕ_2	right lateral bending	left lateral bending	
	x_T	ϕ_3	extension	flexion	
clavicle		θ_1	protraction	retraction	
		θ_2	depression	elevation	
		θ_3	external rotation	internal rotation	
scapula		θ_4	protraction	retraction	
		θ_5	medial rotation	lateral rotation	
		θ_6	backward tilt	forward tilt	
humerus	z_H	θ_7	horizontal adduction	horizontal abduction	DOF
	y_H	θ_8	adduction	abduction	DOF
	x_H	θ_9	external rotation	internal rotation	DOF
ulna	z_U	θ_{10}	elbow flexion	elbow extension	DOF
radius	$- x_R$	θ_{11}	pronation	supination	DOF
hand/disc	z_D	θ_{12}	wrist flexion	wrist extension	DOF

Figure 9: All defined body angles. The second column "Axis" comes from the local coordinate system at each joint, see figure 10D (Figure from Hummel, 2003, p.53).

Each body segment, shoulder, humerus, forearm and hand as well as the disc were equipped with three or more non-collinear markers (Hummel, 2003, p.55). "Five additional markers were used to allow tracking of four virtual joint center markers throughout the throwing trials" (Hummel, 2003, p.55). These estimated joint centres were used as the origins of body fixed Cartesian coordinate systems for each joint centre. Before analysing the position data, it was smoothed with a $10Hz$ Butterworth filter. With the help of a MATLAB algorithm and the knowledge that the distances between local coordinate system origins needed to be constant, Hummel measured or calculated, respectively, the position data of all markers (real and virtual) from 1.25s before until $1s$ after the release of the disc. The MATLAB algorithm worked with the position matrices of the body segments, their gradient and hessian matrix. Euler rotations and hence the angles between the body fixed coordinate systems made the calculation of the DOFs $\theta_7 - \theta_{12}$ possible (Hummel, 2003, p.55ff.). Due to the limited scale of this thesis, the exact way of calculation will not be explained at this point.

To complete the model into a musculoskeletal model, Hummel (2003) used data from Veeger's, Helm's, Woude's, Pronk's and Rozendal's (1991, quoted from Hummel, 2003, p.59) investigations about segment properties of the human body. Veeger et al. (1991) gave segment mass, inertia and centre of mass locations (see table A2 in the appendix). In combination with the estimated angular and linear velocities, Hummel was able to calculate the torques, power and work acting at the angles during the throwing movement.

2.3.2.2 Test Results

The results Hummel presented are based on seven throws from one right-handed male subject. He threw on average with 57% of his maximum launch speed that was measured with a radar gun at $22.4\frac{m}{s}$. Thus, the mean initial speed was $12.7\frac{m}{s}$ with a 0.92 standard deviation and a mean initial spin rate of $46.5\frac{rad}{s}$ (Hummel, 2003, p.60).

2.3.2.2.1 Qualitative Kinematics

Hummel divided the throwing movement into three phases. The first, which she called the wind-up, begins with the left twist of the torso and ends at the maximal torso rotation to the left (See figure 8). The weight of the thrower shifts to the left foot.4 The arm horizontally adducts and the elbow gets slightly flexed to about 50°. The second, the acceleration phase, starts at maximal left twist position and ends with the release of the disc. It is "characterized by sequential uncoiling of the torso and arm segments" (Hummel, 2003, p.60). During this phase, the elbow flexes to 72° at first, before a rapid extension. Follow through, the third phase, begins with the release of the disc and ends at the maximum right twist of the torso. "When the FrisbeeTM is released, the torso is tilted forward, the humerus and torso x axes are nearly aligned, and the arm is externally rotating at the shoulder. The forearm is pronating and the elbow not fully extended. However, the wrist is fully extended" (Hummel, 2003, p.60).

[4] The left foot, caused to the values of torso twist ϕ_1 (see figure 10A), needs to be, viewed from above with 12 o'clock in throwing direction, positioned at four to five o'clock and the right foot at ten to eleven o'clock. However, the positioning of feet and hips is not mentioned by Hummel.

2.3.2.2.2 Quantitative Kinematics

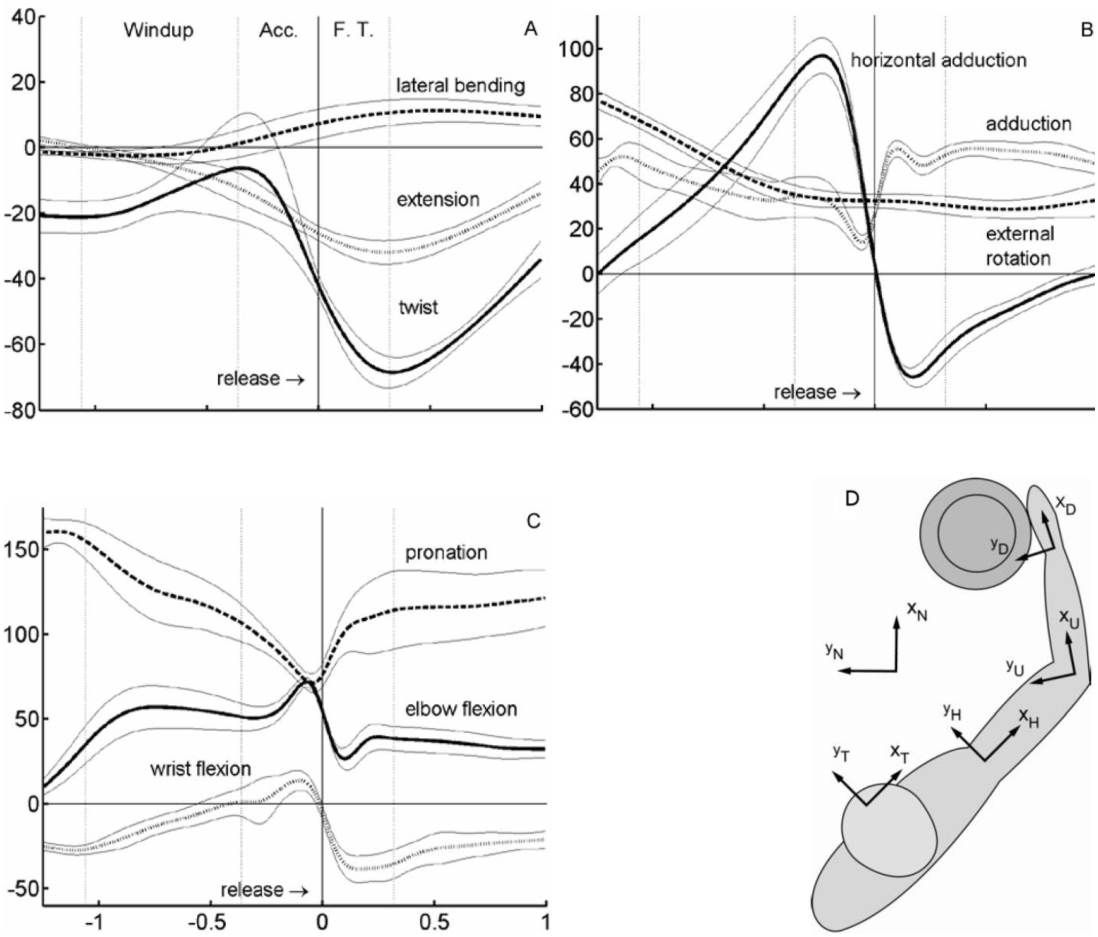

Figure 10: Hummel (2003) plotted the values of joint angles in degrees over time in seconds ($0.5s$ per segment). With the vertical lines she marked windup, acceleration phase, release and follow through. Figure 10A shows the movement of the torso, figure 10B the movement of the humerus and figure 10C the movement of the forearm and wrist. The thinner lines are marking the standard deviation $\pm 1\sigma$ with $n = 7$ throws (Figures from Hummel, 2003, p.62). Figure 10D is a schematic illustration of the thrower before the throw (at $-1.25s$). Here X_N is the direction of the throw. Notice the positioning of the feet explained above (Figure from Hummel, 2003, p.57).

At release in torso twist, horizontal adduction, adduction, elbow flexion, pronation and wrist flexion, a rapid movement is observable. It is conspicuous that the beginning and velocity peak (except of pronation) of these rapid movements in torso twist, horizontal adduction, elbow flexion and wrist flexion follow each other. The widest range occurs in horizontal adduction, which is plotted in Figure 10B. It shows $143°$ of motion in total. The maximum horizontal adduction is $97°$ after windup and the beginning acceleration phase, before the angle decreases with an angular velocity of in peak $-653\,\frac{deg}{s}$ at $-0.04s$. At the release of the disc, it shows $-599\ \frac{deg}{s}$ at 3°. During follow through, the maximum horizontal abduction is 46°. The Elbow flexion stays nearly constant at 50° during wind-up and starts to increase to 72° after the peak in horizontal adduction of the humerus, before decreasing to 27° after release. “At release, the elbow is flexed 57° and has an angular velocity of 431 $\frac{deg}{s}$ in extension” (Hummel, 2003, p.64). The wrist extends at release with 379 $\frac{m}{s}$ (Hummel, 2003, p.63f.). The angular data of torso and the six DOFs is plotted by Hummel (2003) for characteristic points in the following table.

			Angular Displacement				**Angular Velocity**		
			deg				*deg/sec*		*sec*
axis			*max*	*min*	*diff*	*at release*	*at release*	*peak*	*time*
Torso									
z_T	ϕ_1	left twist	-6	-69	63	-42	-155	-168	-0.06
y_T	ϕ_2	right lateral bending	11	1	10	7	15	18	-0.22
x_T	ϕ_3	torso extension	-12	-33	21	-27	-36	-42	-0.13
Humerus									
z_H	θ_7	horizontal adduction	97	-46	143	3	-599	-653	-0.04
y_H	θ_8	adduction	55	14	41	27	434	477	-0.03
x_H	θ_9	external rotation	35	31	4	33	-2	-1	0.02
Ulna									
z_U	θ_{10}	elbow flexion	72	27	45	57	-431	-447	0.03
Radius									
$-x_R$	θ_{11}	pronation	114	71	43	76	224	308	0.06
Hand/disc									
z_D	θ_{12}	wrist flexion	14	-38	52	-5	-379	-397	0.04

Figure 11: Angular displacement and angular velocity. (Figure from Hummel, 2003, p.63)

2.3.2.2.3 Kinetics

Hummel (2003) calculated the kinetics with segment properties, inter alia mass distribution and inertia, based on the model of Veeger et al (1991). She plotted peak joint torques and peak power as well as torque, power and work at release as shown in figure 12 (Hummel, 2003, p.65). Not exactly going ahead with her table Hummel calculated the total work done by the arm joints at release as 35J (Hummel, 2003, p.65), which is more than twice of the calculated average kinetic energy of the disc immediately after release. With an initial speed of $12.7\frac{m}{s}$ and an initial spin rate of $46.5\frac{rad}{s}$, she calculated the translational kinetic energy as $14.3J$ and the rotational kinetic energy as $2.5J$ to in sum 16.8J (Hummel, 2003, p.65). The angular velocity of the spinning disc was nearly in line with the (disc fixed) z-axis. The angle of attack was small, Hummel identified it as 0° to 15° (Hummel, 2003, p.65).

			Peak Torque and time		Peak Power and time		*at release* Torque	Power	Work
			N m	*sec*	*W*	*sec*	*N m*	*W*	*J*
Humerus									
z_H	θ_7	horizontal adduction	34	0.08	312	-0.04	-25	115	35
y_H	θ_8	adduction	36	0.05	6.1	-0.74	13	-0.8	4.2
x_H	θ_9	external rotation	-12	0.04	-90	0.03	-4.7	-35	-1.2
Ulna									
z_U	θ_{10}	elbow flexion	17	0.06	-137	0.03	12	-125	-4.0
Radius									
$-x_R$	θ_{11}	pronation	7.7	0.04	-0.5	0.07	4.9	-0.2	-0.2
Hand/disc									
z_D	θ_{12}	wrist flexion	1.4	0.03	11	0.02	1.1	8.0	0.2
								Total	34

Figure 12: Peak joint torques, peak power and their respective times and torque, power and work at release. (Figure from Hummel, 2003, p.65) Hummel defined the elbow extension causing torque in θ_{10} as positive. (Compare figure 13).

Without further explanation, Hummel plotted the torque, power and work distribution as shown in figures 13 and 14.

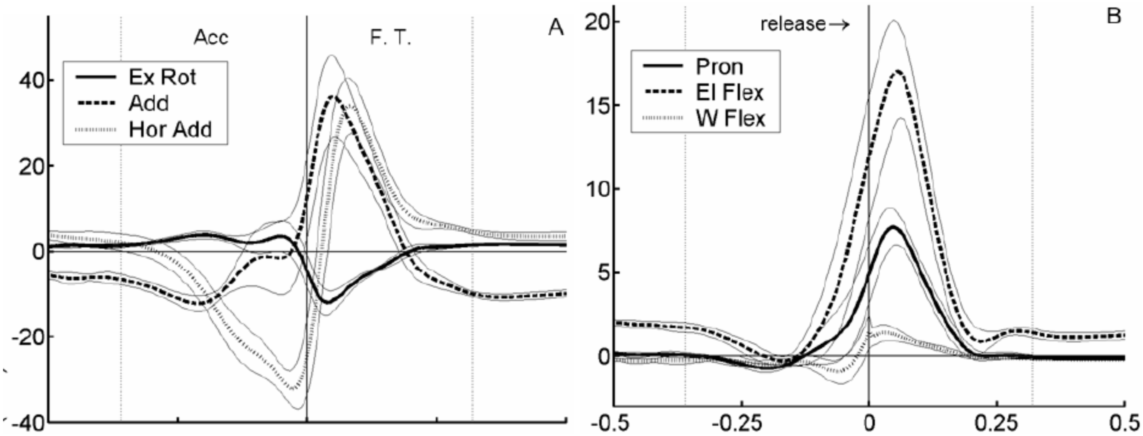

Figure 13: Plotted graphs of torques in Nm over time in $0.25s$ per segment of humerus (14A) and forearm / wrist (14B) respectively during acceleration and follow through (Figures from Hummel, 2003, p.66).

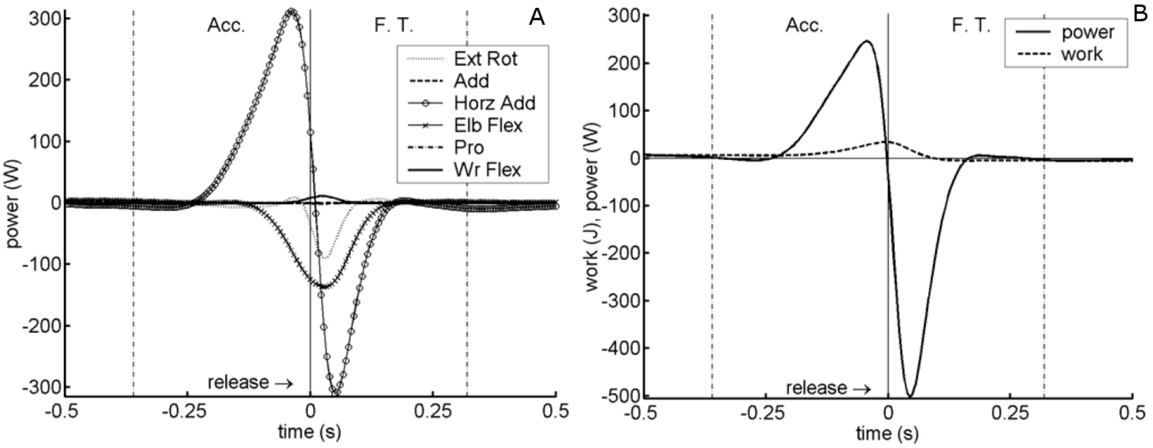

Figure 14: Power of each joint on the left (14A) and power in sum and work on the right (14B) (Figures from Hummel, 2003, p.67). In consequence to the positive definition of the elbow torque, the power of elbow flexion as the product of torque and angular velocity appears negative. Hence, the total positive power before release is smaller than the power of horizontal adduction.

Sarah Hummel introduced the discussion of the biomechanical part of her thesis with the statement that her data was based on the analysis of only one thrower. “It is unknown whether the results are representative of a larger group of Frisbee throwers” (Hummel, 2003, p.67).

Further, she concluded that the Frisbee^{TM} throw partially represents the kinetic chain principle. Although the peak angular velocities appeared consecutively from proximal to distal (compare figure 11), she named the condition that the values of power of all joints in a kinetic chain need to be positive, which was not fulfilled. She found out that the torque of the elbow flexion was positive and therefore the power negative throughout the throw (compare figure 13B). Hummel interprets this negative power as a prevention of a whip-like effect (Hummel, 2003, p.68).

The movement of the elbow joint, which did not fully extend throughout the throw, Hummel interprets as subconscious protection mechanism to prevent hyperextension and injury respectively of the elbow (Hummel, 2003, p.69).

The horizontal abduction was identified as the predominant factor of producing power for the translational movement. This is why Hummel recommended increasing the power at horizontal abduction instead of focussing the wrist snap to novice throwers (Hummel, 2003, p.69).

Finally, Hummel mentioned the change of standard deviations over the throw. With a look at figure 10, it is discernible that the standard deviation varies throughout the throw, but it gets small in the direct surrounding of the release moment for all DOFs. Hummel interprets this as sign for “a preferred configuration by the thrower, even at submaximal effort” (Hummel, 2003. P.70).

2.3.3 Kei Sasakawa and Shinji Sakurai (2008)

2.3.3.1 Setup of Research

A research with the title „Biomechanical analysis of the sidearm throwing motion for distance of a flying disc: A comparison of skilled and unskilled Ultimate players" was published by Kei Sasakawa and Shinji Sakurai in June 2008 (Sasakawa, Sakurai, 2008). The Japanese scientists filmed two groups of throwers while throwing a standard Ultimate Frisbee disc for distance with two synchronized $250Hz$ high speed video cameras from a frontal underneath position. The upper limb and the disc were in the field of both cameras during the whole throw (The throwers stood on a $85cm$ high platform). The first group consisted of ten male players of the local university Ultimate Frisbee team, which has reached the sixth place in Japanese championships. The authors called these group of players with two to four years of experience the skilled group. The second, so called unskilled, group consisted of seven "physically active male students with no experience of any disc sports" (Sasakawa, Sakurai, 2008, p.313). For a better comparability, all subjects were right-handed (Sasakawa, Sakurai, 2008, p.312f.).

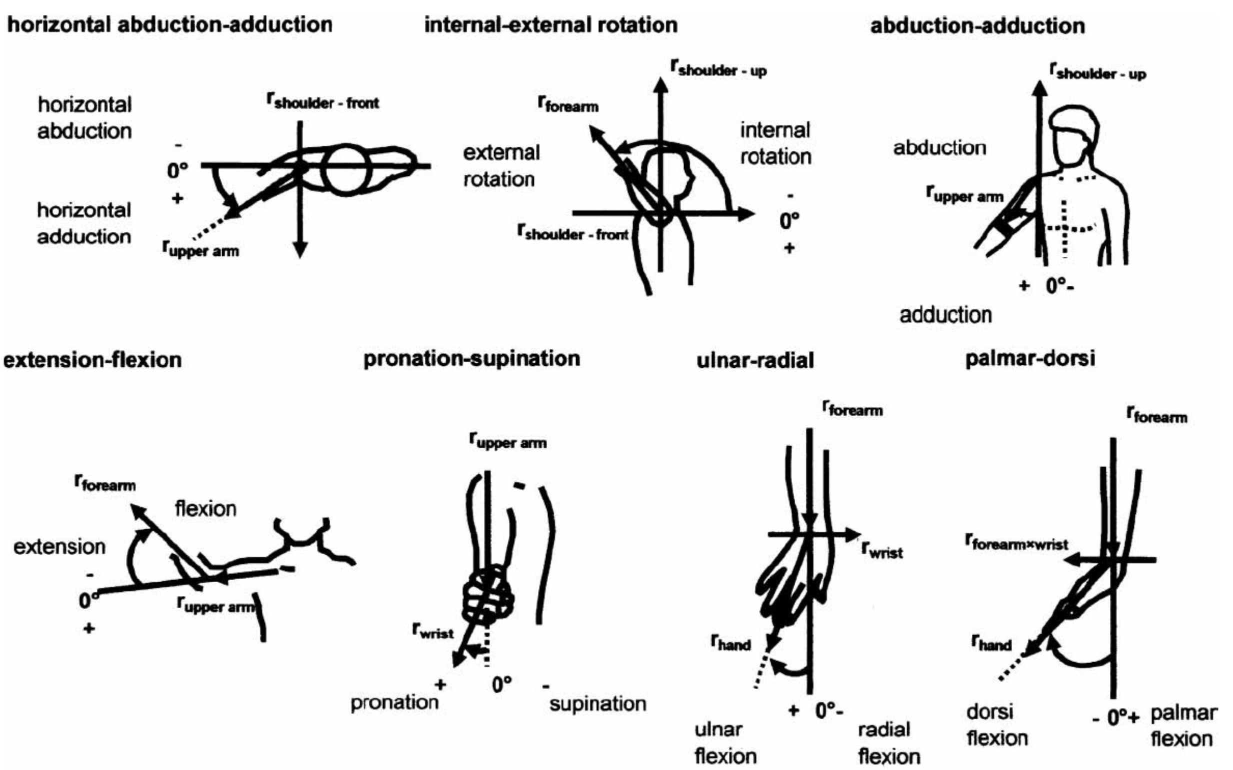

Figure 15: Definition of joint angles (Figure from Sasakawa, Sakurai, 2008, p.316)

Figure 15 shows the definition of joint angles Sasakawa and Sakurai chose for their estimations. They neglected the movement of the upper body and focused on the throwing limb. They named seven observed joint angles. In comparison to the DOFs Hummel (2003) used, they added ulnar / radial flexion of the wrist. The furthest and straightest of ten throws per participant was chosen to be analysed.

The throwing arm of the subjects was equipped with markers at “appropriate anatomical land-marks” (Sasakawa, Sakurai, 2008, p.314) as well as several reference sticks ($15cm$ length, $30g$ weight) on the shoulder (three vertical to each other), the forearm (two in line), the wrist (two in line) and the hand (one in direction of the fingers and two vertical to it in line) (see figure A3 in the appen-dix). The disc was equipped with three non-collinear markers including the disc centre (Sasakawa, Sakurai, 2008, p.314).

2.3.3.2 Test Results

	Skilled	Unskilled
Throwing distance (m)	51.4 ± 6.6*	30.0 ± 7.6
Initial velocity (m/s)	21.7 ± 1.7	20.7 ± 2.5
Spin rate (rps)	12.9 ± 1.3*	9.7 ± 1.3
Angle of attack (°)	0.2 ± 1.1	1.0 ± 3.6
Pitch angle (°)	12.8 ± 3.3	14.2 ± 5.3
Roll angle (°)	− 10.4 ± 9.7	− 14.1 ± 13.2

*Figure 16: Statistical results of mean initial values. The added number is the standard deviation. The symbol * shows a significant difference between skilled and unskilled throwers according to Sasakawa and Sakurai. (Figure from Sasakawa, Sakurai, 2008, p.317)*

The mean initial values of the throws in figure 16 show significant differences between skilled and unskilled players in throwing distance and spin rate. The initial velocity and the orientation angles are not significantly different. The standard deviations tend to be smaller in the group of skilled play-ers, but only the standard deviation of the angle of attack is significantly smaller (Sasakawa, Sakurai, 2008, p.316). The angle of attack of the skilled throwers is not significantly smaller but relatively constant near to 0°. Sasakawa and Sakurai interpreted this fact as a try to throw with low drag by the skilled players. “Skilled throwers were thus considered to achieve longer throwing distances despite almost the same initial velocity as unskilled throwers, because the skilled throwers threw the disc at faster spin rate and smaller angle of attack, minimizing air resistance to the disc” (Sasakawa, Sakurai, 2008, p.319). Sasakawa and Sakurai refer to Potts and Crowther (2002) when explaining the negative initial roll angle as a preventative compensation of precessional effects (Sasakawa, Sakurai, 2008, p.319).

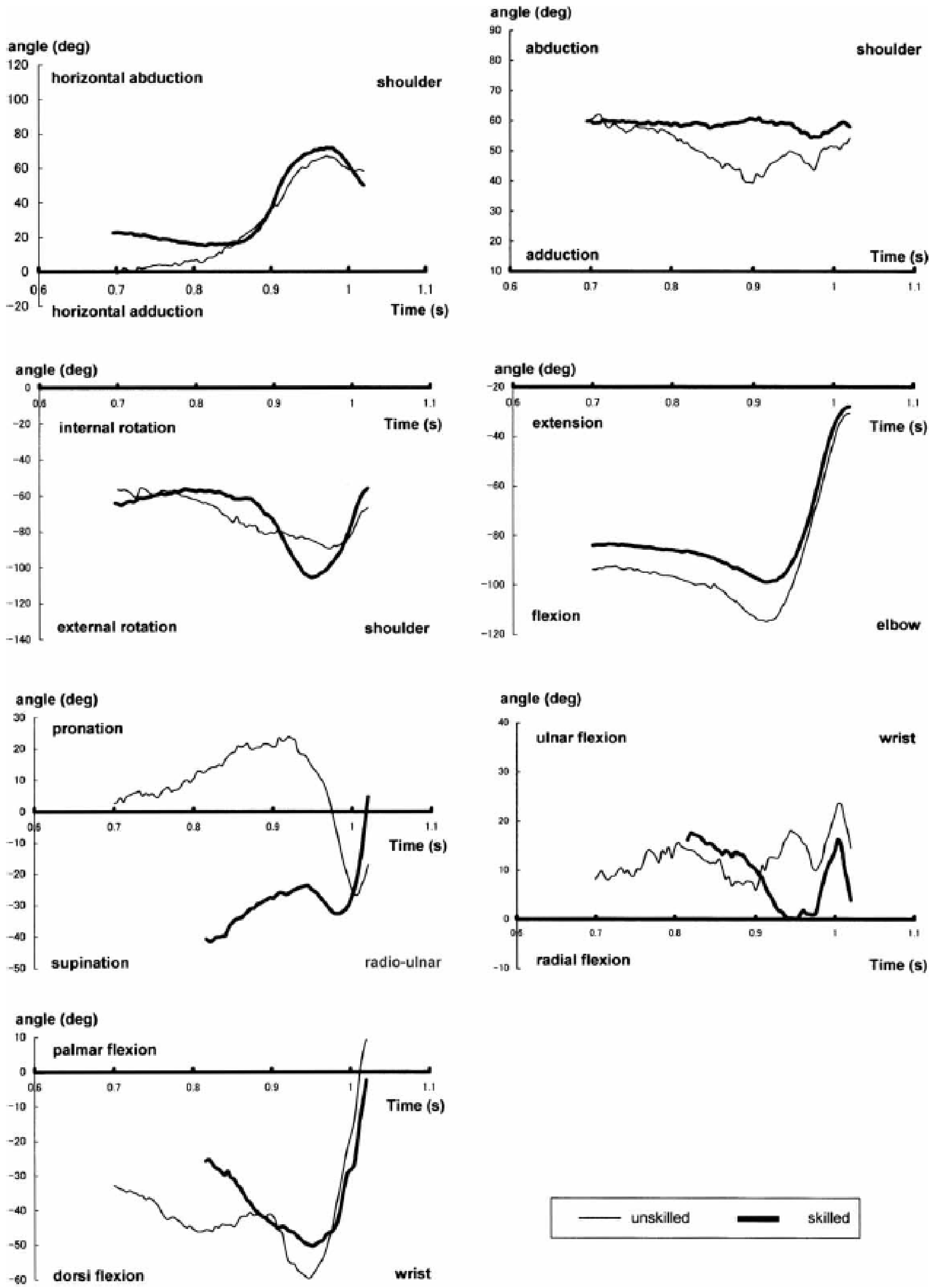

Figure 17: Comparison of throwing motion between skilled and unskilled throwers. Release of the disc is at $1s$. *(Figure from Sasakawa, Sakurai, 2008, p.318)*

In figure 17 the mean angle values of the skilled and the mean angle values of the unskilled players are plotted. The throwing motion curves show basically the same characteristics for skilled and unskilled players with exceptions in e.g. the pronated acceleration and supination immediately before release of the disc. The skilled players accelerate the disc in a supination of 40° to 30° before rapidly pronating the forearm while the disc is released. Less considerable differences were observed in adduction of the humerus and ulnar/radial deviation of the wrist. While the abduction angle of skilled players is nearly constant at 60°, it decreases to 40° and increases to almost 60° again throughout the throwing motion of the unskilled participants (Sasakawa, Sakurai, 2008, p.317).

Sasakawa and Sakurai compared the throwing motion of the forehand Frisbee™ throw with the pitch throw in Baseball. Based on Papas et al. (1985), Dillman et al. (1993), Sakurai et al. (1993) and Fleisig et al. (1995) (all quoted by Sasakawa, Sakurai, 2008, p.319) in an overhand throwing motion, internal rotation of the shoulder, extension of the elbow, pronation of the forearm, ulnar deviation and palmar flexion are observable. However, all these partial movements were performed by the skilled Ultimate Frisbee players throwing with the forehand (Sasakawa, Sakurai, 2008, p.320) (compare figure 17).

The "snap" motion, which is often emphasized by coaches and in textbooks for beginners of Ultimate Frisbee™, seems to be visible in the rapid movements immediately before the release of the disc in figure 17. It is produced "by a sequence of motion comprising pronation after supination in the forearm, and palmar flexion and ulnar deviation after dorsi flexion and radial deviation just before disc release" (Sasakawa, Sakurai, 2008, p.320). This "snap" motion is often said to produce spin. Figure 18 shows values in supplementing to the curves in figure 17.

	Skilled		Unskilled	
Angular position	MER (°)	DRL (°)	MER (°)	DRL (°)
Shoulder				
Horizontal abduction	68.5 ± 14.7	62.1 ± 14.5	63.5 ± 27.6	59.2 ± 24.4
External rotation	−107.1 ± 19.7	−74.3 ± 13.3	−91.7 ± 20.7	−80.7 ± 24.7
Abduction	59.1 ± 9.7	57.6 ± 7.8	44.3 ± 19.3	51.3 ± 12.7
Elbow				
Flexion	−88.7 ± 7.4	−36.2 ± 6.5	−79.1 ± 20.2	−42.3 ± 14.5
Forearm				
Supination	−26.2 ± 22.7*	−26.7 ± 13.3	0.3 ± 14.2#	−25.4 ± 18.9
Wrist				
Ulnar deviation	0.7 ± 9.6*	14.0 ± 6.1*	16.1 ± 16.8	21.4 ± 6.6
Dorsi flexion	−50.0 ± 9.5	−28.3 ± 8.4*	−46.1 ± 13.0	−18.6 ± 8.8
Time (s)	0.948 ± 0.009	1.000	0.972 ± 0.019	1.000

*Figure 18: Joint angles at maximum external rotation (MER) and disc release (DRL) in degrees. *: Significant difference to the unskilled group. #: A negative value means a pronated position. (Figure from Sasakawa and Sakurai, 2008, p.319)*

3. Discussion

3.1 A Comparison of Backhand and Forehand Throws

The aim of replacing the release point of the throwing motion sideways to avoid contact with a direct opponent player results in the fact that an Ultimate player needs to be able to throw the disc on several ways. The backhand motion is the kind of throw the most layman choose and which is mostly used. Disc Golf players usually use the backhand throw but even in this field, just as in most kinds of disc sports, both ways of throwing are used. Therefore, both kinds of throwing motion should be in the focus of interest.

The skilled Ultimate players in Sasakawa's and Sakurai's and the investigated thrower in Hummel's research threw with a low, but positive, angle of attack. They probably did not actually think about the physical background, but their experience taught them to do so. This way, they seem to achieve a beneficial drag and lift ratio (compare figure 3). At a low angle of attack, the pitching moment stays controllable as well. It has been noticed that a high angular velocity in spin is another good remedy to control the pitching moment and therefore, the precessional roll rate. To oppose this roll rate, a disc is thrown with a compensating bank to the other side. In a normal pass on an Ultimate field or a Frisbee^{TM} throw for distance in Disc Golf, the angle of attack is low. Hence, the pitching rate is negative and thus, a forehand must be thrown with a negative and a backhand throw must be thrown with a positive roll angle in order to fly straight.

The mean spin rate of the backhand throws in Hummel's investigations was $46.5\,\frac{rad}{s}$, the mean spin rate of the skilled group throwing a forehand was $12.9\,\frac{rad}{s}$. In consequence, the precessional roll rate of sidearm throws should be higher at a factor of three to four. It is unknown whether the difference in spin rate between backhand and forehand is caused by the experience of each thrower or the different throwing techniques, which were used. A factor of more than 350% difference in spin rate seems to be no coincidence. At this point, there is a lack of investigation in literature. It could be, for example, interesting to investigate the differences in precession between a backhand and forehand throw.

Due to body constitution, the two main ways of throwing a disc show considerable differences (Compare figure 10 and 17). As expected, the characteristics of horizontal adduction and palmar / dorsal flexion of the wrist show a roughly opposite curve. However, regarding the horizontal adduction, the point of release appears immediately after the second change of direction in the forehand throw, while it appears somewhere midways between the changes of direction in the backhand throw.

Adduction and rotation of the humerus show no resemblance. The elbow flexion curve shows a roughly similar profile but with a far less pronounced flexion before the rapid extension whilst the disc accelerates. The not yet clear role of pronation and supination, which is discussed very shortly in the researches at hand, needs further investigation. It is not possible to make statements about similarities with the data at hand. Due to the small range of data and the small number of investigations, the tendencies given here should also be considered with caution.

In general, a comparison between a backhand and a forehand throw based on the studies at hand is difficult to make due to the different methods that were used. The investigations of Hummel (2003) are presented in a much more detailed form than these of Sasakawa and Sakurai (2008). A calculation of torques, power or work distributions was not made for the forehand throw.

3.2 A Comparison between Pesch's and Hummel's Investigations about the Backhand Throw

Due to various arguments, Pesch's research is not discussed in detail. It has never been published and therefore not proofread to the necessary extent. The methods of his investigation produced several unpreventable mistakes explained above. He orientated a high number of conclusions towards the ideal value, which was taken from the one furthest throw during his examination. The kind of investigation he did is not to be neglected but needs further proof by means of similar investigations, which try to minimize the mistakes in measuring and marking. After considerable searching it appears the Master Thesis from Sarah Hummel has not been published either but quoted by several authors (e.g. Potts (2005)) from later published researches. Furthermore, Hummel and Hubbard published the main results of the biomechanical researches in Hummel's Master Thesis two years earlier (2001) on the 8th International Symposium on Computer Simulation in Biomechanics in Milano, Italy.

A comparison between the two research projects examining the backhand throw from Pesch (1999) and Hummel and Hubbard (2001) or Hummel (2003) respectively is difficult to make due to different methods and different focuses of each research. However, several details, such as the curve of horizontal shoulder and elbow movement (see figure 10 in 2.3.2.2.2 and figure A1 in the appendix), show roughly similar results. Pesch did not mention the kinetic chain, but in figure A1 it is observable that at least the peak angular velocity of the elbow follows the peak angular velocity of the shoulder.

Pesch (1999) plotted the movement of the body's centre of mass over time and showed a decrease of $15cm$ during wind up and acceleration with a turning back immediately before release (See figure A2 in the appendix). Hummel (2003) did not mention any movements of the body's centre of mass.

Another example of partial movement, which was investigated in only one of both studies, is the moment of placing the foot in throwing motion. While Hummel neglected the movement of the lower body, Pesch orientated his analysis on this moment. While Hummel estimated torques, power and work of each body segment, Pesch described the data he found. Thus, both of them did not describe the whole throwing movement but made several necessary simplifications to enable the description of the throw. At this point, there is potential for future investigations.

3.3 General Discussion

Sasakawa and Sakurai (2008) could not proof this fact, but regarding figure 17, the main difference in motion between skilled and unskilled throwers was in the movement of the forearm and wrist. The main difference in initial flight conditions was in the lower spin rate. These circumstances seem to show that a beginner should try to adapt his forearm movement to achieve higher spin rates and thus further throwing ranges. Sasakawa and Sakurai concluded: "The critical factor in throwing a disc for distance is not initial speed, but spin rate" (Sasakawa, Sakurai, 2008, p.320). Opposite to this statement, Hummel (2003) pointed out that the horizontal abduction of the humerus was predominant in power distributions and should therefore be primarily in the focus of beginning throwers. These two conclusions remain assumptions and need to be proofed. Hung, Kaminski, Fineman, Monroe and Gentile (2008) investigated the adaption and organisation of arm movements during a learning period of 1300 trials of a Frisbee™ throw and found out that the path of the arm joints got more constant over time. Yang and Scholz (2005) made similar investigations, which led to the main conclusion that "overall joint configuration variability decreased with practice" (Yang, Scholz, 2005, p.153).

Sarah Hummel (2003, p.64) identified the horizontal adduction as the predominant factor in power contribution of a backhand Frisbee™ throw. The power acting at release was measured as $115W$. At release, it has already decreased because $0.04s$ earlier, Hummel (2003, p.63) measured the peak value as $312W$. At release, Hummel (2003, p.65) calculated the torque as $M = -25Nm$ and the angular velocity as $\omega = -599\frac{deg}{s} \approx -10.45\frac{rad}{s}$. Due to $P = \vec{M} \cdot \vec{\omega} = M \cdot \omega \cdot \cos(\pi)$ with π as the angle between $\vec{M}$ and $\vec{\omega}$, the power would be $P \approx 261.36W \cdot \cos(\pi)$. If π is close to $0°$, which can be assumed because the attitude of the velocity vector and the torque, which produces this velocity vector, should be in line, it will be $\cos(\pi) \approx 1$. Hummel (2003, p.65) named the power at release as $115W$. This difference could be caused by a problem in measurement, but it could also be due to a significant displacement of the attitudes of torque and angular velocity vectors. Probably, because of the limited scale of her thesis, Hummel (2003) did not mention the way she calculated the values for torques, power and work in detail.

However, the calculated power in horizontal adduction is positive due to the negative torque Hummel (2003) named. In case of elbow extension, Hummel gave a positive torque for the moment of release, in spite of a negative angular velocity, which represents an elbow extension. As a mathematical consequence, she received a negative power for elbow flexion (compare figures 11 and 12). Later in her discussion (Hummel, 2003, p.68), Hummel named this negative power as the reason why the Frisbee™ throw does not fully represent the kinetic chain principle. She named two conditions, which needed to be fulfilled (Hummel, 2003, p.68). The first was the sequence of peak angular velocities from proximal to distal, which could, with the exception of pronation, be declared as fulfilled in the case of a backhand throw (compare figure 11). The second condition Hummel named was that all involved joint powers needed to be positive. This condition, which could not be proofed by literature research, is - caused by the negative power in elbow flexion - not fulfilled. Hence, Hummel concluded that the backhand throw does not fully represent a kinetic chain.

Moreover, Hummel interpreted the negative power in elbow flexion as a deceleration of the forearm, which would prevent a whip-like effect (Hummel, 2003, p.68). An unanswered question in this context is why she defined the torque at release, of for example horizontal adduction, as negative and the torque of elbow flexion as positive. If Hummel had defined this torque negative, the power would have been positive and the kinetic chain principle Hummel declared would have been fulfilled. In the hypothetical case that the torque of elbow flexion was negative, the power at release would be $125W$ and a little higher than the power of horizontal abduction, which was named as $115W$. The statement that the horizontal abduction is the predominant factor in power distribution would not be that clear, as it appears in Hummel's Thesis. However, regarding the development of power over time (compare figure 14), the sum of acting power over time until release would still be higher in horizontal adduction, even if the power of elbow flexion was positive. The distribution of the work, which is done at each joint, would also still be predominant at the horizontal adduction, even with inversed signs at the elbow flexion.

As mentioned above, the power of horizontal abduction decreases immediately before the release of the disc. This could refer to the third of Newton's laws and the movement of more distal limb segments, especially the elbow extension, which starts its rapid acceleration shortly after the horizontal acceleration in the shoulder. The biomechanical principle of optimizing the temporal coordination of partial impulses represents a variation of the first condition Hummel named for the kinetic chain principle. The biomechanical principle of partial impulses does not refer to angular velocity but to translational movement. Following Hummel's results, the times of peak angular velocities appear in a sequence from proximal to distal. The angular velocities do not directly determine the translational velocities of the concerning joints, which are not given, but with a qualitative interpretation of the joint angular velocities and their directions this biomechanical principle is likely to be fulfilled.

The principle of initial force, which states that a muscle will produce a higher force impact if it is lengthened immediately before its contraction, seems to be used by horizontal adduction during wind-up. The effect is caused by biological mechanisms, e.g. the reflex of a muscle when being stretched or the elastic energy in muscles and tendons respectively. In the backhand Frisbee^{TM} throw motion (compare figure 10) in torso twist, horizontal adduction and wrist flexion, a clear pre-streching during wind up or shortly after is observable. In elbow flexion, a kind of pre-stretching is observable as well, but this kind is much less clear and could also be interpreted otherwise.

The absolute value of the torque of horizontal adduction increases constantly from $0.25s$ until round about $0.04s$ before release (compare figure 13). $0.25s$ before release is the time when the humerus returns its direction. This can be interpreted as a use of the principle of the optimal acceleration distance. The force impact increases because a force, here induced by a muscle, can act over a longer time. The aim of this principle is a high velocity of a projectile at release. The longer and the higher a force or torque is acting, the higher is the impulse of the projectile at release. Due to joint leverage and optimal lengthening and tiredness of muscles and tendons, the acceleration distance has a biological border. Hence, especially for throwing or jumping movements - imagine a jump without, with optimal or with very deep bending of the knees – there exists an optimal acceleration distance. The force impact of a partial movement can be taken by the surface integral of the torque from the moment the torque begins to act in the intended direction until release. Regarding figure 13, the horizontal abduction has obviously the highest force impact at release. The second highest force impact seems to be given by elbow flexion, but this acts, according to the interpretation of Hummel, against the throwing direction. If, as discussed above, the elbow flexion torque had had a negative sign, its force impact would have affected the throwing movement by an additional acceleration of the forearm. Taking into consideration the course of torque over time, its value decreases to zero at about $0.22s$ and starts to increase again by taking on positive values at about $0.15s$ before release, it appears that the aim is not to reach the highest possible velocity. This time period goes roughly ahead with the phase of increasing elbow flexion while the humerus is starting to accelerate horizontally. This not generated possible velocity of the forearm is probably caused by the intention to control the attitude of the disc at release. It can oftentimes be observed that a novice thrower loses control when trying to throw the disc with too much kinetic energy.

4. Conclusion

4.1 Summary

Literature about the biomechanical aspects of throwing a Frisbee^{TM} was searched. Four scientific works were found but only three of them could be accessed. Investigations regarding the physics of a flying disc are available in a higher number.

The general physical background of a Frisbee^{TM} flight, in particular drag, lift and gyrodynamics as well as the trajectory of a flying disc, was presented. It was found out that a thrower should achieve a relatively low angle of attack and a preventative roll angle depending on the flight trajectory he aims at. To keep the precessional roll rate controllable, a high spin rate in the throwing movement should be produced.

An overview over the used methods in flying disc science was given in part 2.2. Several studies about the physics of flying discs were carried out with wind tunnel measurements. The most embracing publication is the doctoral dissertation from Potts (2005). Furthermore, several computer models tried to describe the flight of a flying disc. Mostly calculated with a MATLAB algorithm, several coefficients were used for this description. Lorenz (2005) tried to widen the knowledge about Frisbee physics with on board measurements. The high speed camera is the primarily used instrument when investigating the biomechanics of a Frisbee throw.

Part 2.3 presented biomechanical research. Pesch (1999) took a look at the opinion and the techniques German High level Ultimate players used for long range backhand throws. However, his work has not been published and the way of investigation included several unpreventable mistakes. The research project of Hummel and Hubbard (2001) or Hummel (2003), respectively, payed attention to the backhand throw and included calculations of torques, power and work acting at the focussed joints. Their calculations were based on high speed video data of one high level US Ultimate player throwing with medium effort. The investigations of Sasakawa and Sakurai (2008) compared high speed video data of the sidearm throwing movement of skilled and unskilled throwers and gave a quantitative description of the throwing movement. A comparison between the different studies as well as between the two types of throwing movement has been difficult and is, due to the small number of sources, only partly meaningful. It can be concluded that the Frisbee^{TM} throw partially fulfil several biomechanical principles. The horizontal abduction seems to be the main part giving power to the disc in a backhand throw. The role of elbow flexion is not clear. This and several other points need to be investigated in the future.

4.2 Future Work

There is a high number of ways to throw a flying disc but a very small number of investigations done on this field. Hummel and Hubbard (2001) and Hummel (2003) investigated the backhand throwing movement but without a description of the lower body. Hummel (2003, p.67) herself remarked that it is unknown whether the results of her investigations are representative for a larger group of throwers. The analysis Hummel (2003) made was based on only one thrower, but in comparison to the publication of Sasakawa and Sakurai (2008) much more detailed. Saskawa and Sakurai (2008) did not name velocities or calculated torques, power or work. Therefore, it is still not possible to make scientific statements in relation to kinetics or the biomechanical principles in throwing a disc with the forehand. At this point, there is a high potential for future work.

What happens in the lower body during the throwing movement has only been investigated by Pesch (1999) very shortly. A comparison between skilled and unskilled Ultimate players throwing a backhand has not been researched yet. A comparison between players of Ultimate, Disc Golf or other disc sports has not been made yet. A comparison to other throwing projectiles could also be a field of future investigation. There are a lot of grey fields on the land map of research in the biomechanics of a Frisbee™ throw.

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5. Appendix

Table A1 lists all researches about Frisbee™ flight, which could be found, ore are quoted be the publications at hand.

Author	**Title**	**Year**	**Quotation From**	**Overview**
Stilley, Carstens	Adaption of Frisbee Flight Principle to Delivery of Special Ordnance *in AIAA Paper No. 72-982*	1972	Potts, 2005, p.44 Hummel, 2003, p.3	Wind Tunnel Investigations for axi-symmetric flying discs, different shapes. Investigation of the influence of spin on the aerodynamics. It was found as negligible.
Contreo	An 8 DOF lumped-parameter model of shoulder motion *Master Thesis, California State University*	1980	Hummel, 2003, p.2	Comparison of backhand and forehand Throws. Focus on force contribution of body segments.
Lazzara, Schweitzer, Toscano	Design and Testing of a Frisbee Wind Tunnel Balance *unpublished Report*	1980	Potts, 2005, p.45	Wind Tunnel measurement. Lift and drag was measured. Influence of spin. Spin generates a small lift component.
Nakamura, Fukamachi	Visualization of the flow past a Frisbee *in Fluid Dynamics Research 7.1 (1991):31-35*	1991	at hand	Low speed wind tunnel visualizations show two vortices past the Frisbee.
Ali	Aerodynamics of Rotating Disc Wings *in Division of Aerospace Engineering, The Manchester School of Angineering, University of Manchester*	1998	Potts, 2005, p.45	Wind Tunnel measurement. Lift and Drag.
Pesch	Technikanalyse des weiten Rückhandwurfes im Ultimate Frisbee - Eine empirische Untersuchung zur Strukturierung und zur	1999	at hand	1. Description of Ultimate, Rules, Tactics etc. / Physics of the disc / 2. Method: survey of ten high level players and video data analysis of these players while throwing. Idea: getting the theoretical opinion about the best movement in throwing a huck from the best players and prove that by filming them.

	Identifikation von Einflußgrößen mit Ableitung von konkreten Bewegungsanweisungen *Diplomarbeit - Johannes Gutenberg Universität Mainz*			Description of throw with several data sets. (Morphologic, angle, flight data...) 3. Biomechanical factors involved in throwing for different phases of motion 4. Concrete advise for throwing motion
Mitchell	The Aerodynamic Response of Airborne Discs *Master Thesis, University of Nevada, USA*	1999	Potts, 2005, p.45	Wind Tunnel. Investigation of Lift/Drag Ratio and Reynolds number, each regarding to the angle of attack. He found a strong connection between angle of attack and Reynolds number.
Yasuda (in Japanese)	Flight- and Aerodynamic Characteristics of a Flying Disk *in Japanese Soc. Aero. Space Sci., Vol. 47, No.547, pp 16-22*	1999	Potts, 2005, p.47 Hummel, 2003, p.3	Wind Tunnel. Measurement of Drag and Lift.
Hubbard, Hummel	Simulation of Frisbee Flight *In Proceedings of the 5 th Conference on Mathematics and Computers in Sport, University of Technology, Sydney, Australia*	2000 June	at hand	Flight equations compared with high speed camera results. Good estimation for flights about 2m; longer flights are very sensitive to aerodynamic coefficients
Potts, Crowther	The flow over a rotating Disc-Wing *in RAeS Aerodynamics Research Conference Proc., London, UK.*	2000 April	at hand	Wind Tunnel Measurements. Reynolds number has negligible Influence. Spin Rate influence is low, but shows some decrease in rolling moment. Topology of flow is visualised.
Potts, Crowther	Visualisation of the Flow Over a Disc-Wing *In Proc. of the Ninth International Symposium on Flow Visualization, Edinburgh, UK.*	2000 August	at hand	Wind Tunnel visualisation technics for understanding 3D structure of flow around a flying disc.
Higuchi, Goto, Hiramoto, Meisel	Rotating Flying Disks and Formation of Trailing Vortices *in AIAA 2000-4001, 18th AIAA Applied Aero. Conf., Denver, USA*	2000 August	Potts, 2005, p.46	Wind Tunnel. Laser flow visualisation. Particle image velocimetry. Disc Golf disc. Focus on vortices.

Hummel, Hubbard	A Musculoskeletal Model for Backhand Throws *in 8th International Symposium on Computer Simulation in Biomechanics, Politecnico di Milano, Milan, Italy*	2001	at hand	High speed cameras were used to estimate joint positions and angular velocities during a throw.
Potts, Crowther	Application of Flow Control to a Disc Wing UAV* *unmanned air vehicle *in 16th Bristol UAV Systems Conference*	2001	at hand	Experimental investigation for flow of a non-spinning disc in wind tunnel. Comparing a slick disc, a disc with turbulence stripes (as used in Ultimate) and a disc with fences.
Potts, Crowther	Flight Control of a spin stabilised axi-symmetric disc-wing *in 39th Aerospace sciences meeting & exhibit, Reno, Nevada, USA*	2001	at hand	Very similar to Potts, Crowther (2001) on Bristol conference.
Hummel, Hubbard	Identification of Frisbee aerodynamic coefficients using flight data *In 4th International Conference on the Engineering of Sport, Kyoto, Japan*	2002	at hand	Matching (filmed) flight data with predicted data from simulation model (computer). H and H estimated ten aerodynamic coefficients with an optimization algorithm. Performance of results depends heavily on accurate measured data. Results are comparable to wind tunnel results from literature.
Motoyama	The Physics of Flying Discs *in unknown*	2002	at hand	Theoretical mathematical or physical description of Frisbee flight: Physical Background Gyroscopic Inertia and Bernoulli and aerodynamics and gyrodynamics.
Potts, Crowther	Frisbee Aerodynamics in 20th AIAA Applied Aerodynamics Conference 2002, St. Louis, Missiouri	2002 June	at hand	Short literature overview; Airflow and aerodynamic investigation of non spinning (see 2001) and spinning discs. Spin effects lift, drag, pitching- and rolling moment small but measurable. This is viewable in flow vortices.
Hummel	Frisbee Flight Simulation and Throw Biomechanics *Thesis Master of Science - University of California*	2003	at hand	1. Dynamics: Aerofoil and Gyroscope 2. Mathematical Model compared with high speed video data as an alternative to wind tunnel test data. The coefficients in the Model are

				calculated with an iterative MATLAB algorithm. 3. Biomechanical Model for Backhand Throw. Angular velocities, positions, torques and power distributions are estimated by high speed video Data.
Lorenz	Flight and Attitude Dynamics Measurements of an Instrumented Frisbee *in Measurement Science and Technology*	2005	at hand	In flight measurement of several parameters with accelerometers and other sensors placed underneath of a flying disc.
Morrison	The Physics of Frisbees *in Electronic Journal of classical Mechanics and Relativity*	2005	at hand	Mathematical/ physical description of Frisbee flight. Theoretical description of aerodynamics and gyrodynamics.
Yang, Scholz	Learning a throwing task is associated with differential changes in the use of motor abundance *in Experimental brain research*	2005	at hand	High speed camera. Filming the changes in synergy of joint motions. Learning throwing movement.
Potts	Disc-wing Aerodynamics *Doctor Thesis - University of Manchester*	2005	at hand	Aim: find a design for the best flying disc. Pitching moment by unspinning discs - gyroscopic effects chances this moment in a roll moment. Aerodynamic characteristics of several disc designs. Characteristics of flow around a disc. MATLAB Model of the trajectory of a flying disc.
Lorenz	Spinning Flight: Dynamics of Frisbees, Boomerangs, Samaras, and Skipping Stones. *New York: Springer*	2006	at hand	Popular scientific summary of several researches. One chapter about Frisbee™ flight. No additional investigation.
Crowther, Potts	Simulation of a spin-stabilised sports disc *in Sports Engineering*	2007	at hand	A six degree of freedom mathematical model of a rotating disc-wing.
Hubbard, Cheng	Optimal discus trajectories *in Journal of Biomechanics*	2007	at hand	Mathematical model for discus flight.

Hung, Kaminski, Fineman, Monroe, Gentile	Learning a multi-joint throwing task: a morphometric analysis of skill development *in Experimental brain research*	2008	at hand	Qualitative and quantitative description of learning an aimed Frisbee throw. Observation of learning process with high speed cameras.
Sasakawa, Sakurai	Biomechanical analysis of the sidearm throwing motion for distance of a flying disc: A comparison of skilled and unskilled Ultimate players *in Sports Biomechanics*	2008 Sep.	at hand	Forehand: Joint angles of throwing limb were examined. Ten skilled, seven unskilled participants threw for distance. No significant difference in initial speed, but in spin rate and distance. Difference in motion especially in forearm pronation/Supination and smaller in wrist motion.
Lissaman, Hubbard	Maximum Range of Flying Discs *in Procedia Engineering*	2010	at hand	Theoretical comparison of ballistic trajectories with flying disc trajectories. Ascent, glide and flare out. Close look to angle of attack.
Baumback	The Aerodynamics of Frisbee Flight *in UJMM Journal*	2010	at hand	Two dimensional mathematical description of Frisbee^{TM} trajectory.
Koyanagi, Seo, Otha, Ohgi	A computer simulation of the flying disc based on the wind tunnel test data *in Procedia Engineering*	2012	at hand	Theoretical Frisbee^{TM} trajectory model. Compared with wind tunnel data.
Lukes, Hart, Potts, Haake	A CFD Analysis of Flow Around a Disc *in Procedia Engineering*	2014	at hand	Computational fluid dynamics (CFD) analysis with different standard flow models. A few produced unrealistic flow phenomena, but the k-"epsilon" model produced the most suitable data.

Table A2: Mass, inertia position and length of different body segments, based on the data of Veeger et al. (1991) (Table from Hummel, 2003, p.59).

Segment	Mass (kg)	Inertia (I_x) (kg m^2)	Inertia (I_y, I_z) (kg m^2)	% cm position[A]	Length (m)
torso	-	-	-	-	-
clavicle [C]	0.31	-	-	0.50	0.15
scapula [C]	0.69	-	-	-	0.15
humerus	1.72	0.0013	0.0143	0.53	0.35
ulna [D]	0.52	0.0001	0.0041	0.39	0.27
radius [D]	0.52	0.0001	0.0041	0.39	0.27
hand[B]	0.46	0.0002	0.0006	0.40	0.11
disc	0.175	0.00122	0.00122, 0.00235	0.50	0.27

[A] Ratio of segment length measured from proximal end. [B] Hand and disc were treated as one segment until release. [C] Approximation of Veeger torso data (Cote and Hubbard, 2003). [D] Approximation of Veeger forearm data (Lemay and Crago, 1996).

Figure A1: Curves from Pesch's (1999) investigations. Release of disc on both curves is at picture 66. (Figures from Pesch, 1999, p.88)

Left: Horizontal shoulder and elbow movement. The upper curve shows the angles of the elbow over the time in pictures and the lower curve shows the horizontal shoulder movement.

Right: Angular velocity of horizontal shoulder and elbow movement.

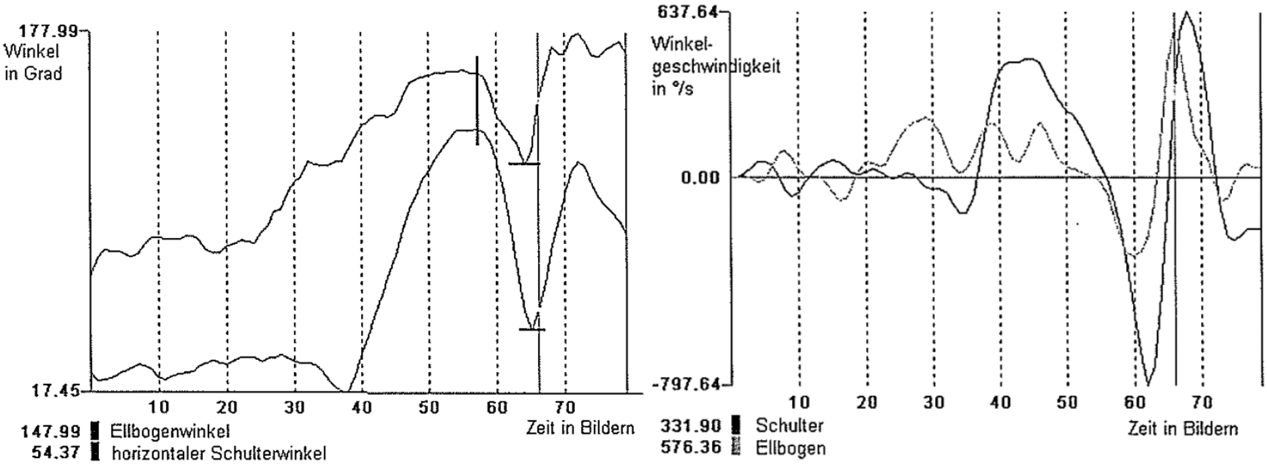


Figure A2: Movement of body's center of mass in Y-direction (left curve) and in Z-direction (Figures from, Pesch, 1999, p.87 and p.89).

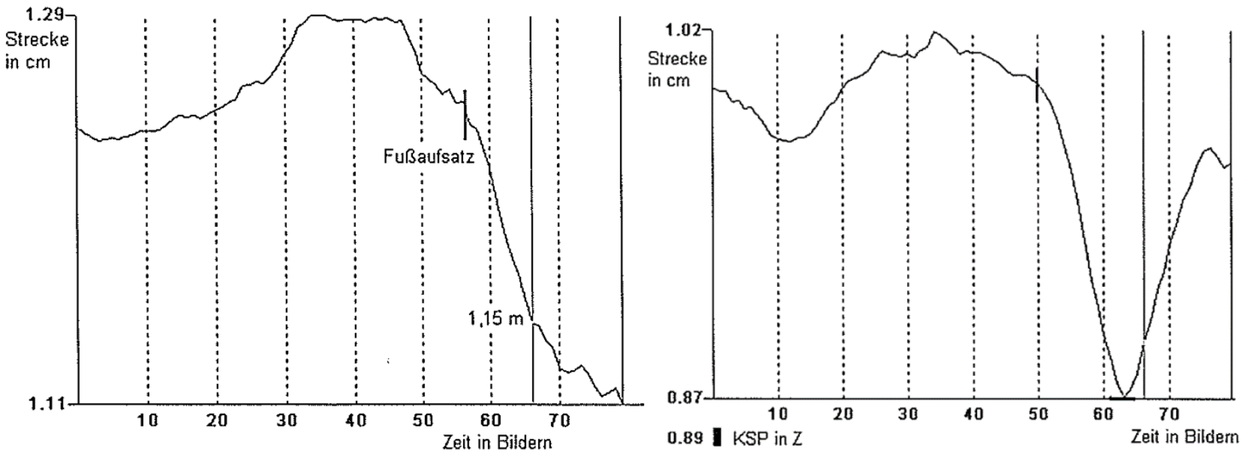
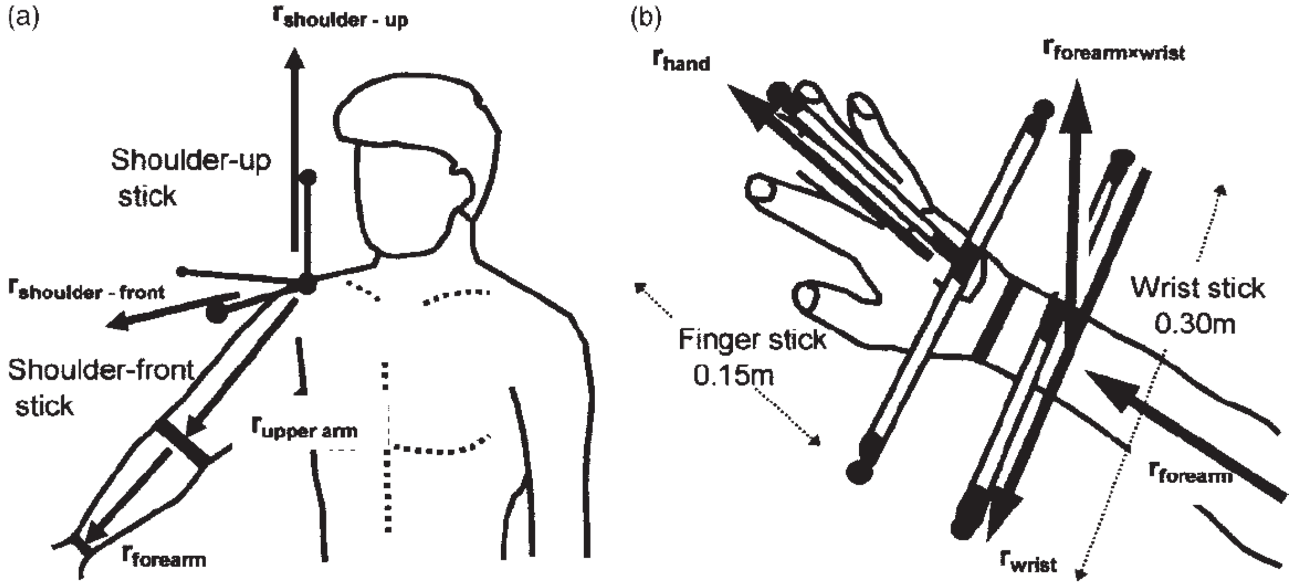


Figure A3: Positioning of marker sticks in the investigations of Sasakawa and Sakurai (Figure from Sasakawa, Sakurai, 2008, p.314).



6. Declaration of Academic Integrity

I hereby confirm that this thesis on “Biomechanical Aspects of Throwing a Frisbee: A Review” is solely my own work and that I have used no sources or aids other than the ones stated. All passages in my thesis for which other sources, including electronic media, have been used, be it direct quotes or content references, have been acknowledged as such and the sources cited.

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(date and signature of student)

I agree to have my thesis checked in order to rule out potential similarities with other works and to have my thesis stored in a database for this purpose.

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(date and signature of student)